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STRENGTH OF COLUMNS UNDER COMBINED BENDING AND THRUST

by

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C. E. 213

Structural Research

August 31, 1951

Sept. 18, 1951

Department of Civil Engineering
and Mechanics
Lehigh University
Bethlehem, Pennsylvania

ATTENTION: Professor Eney and Mr. Beedle

Gentlemen:

The following report is presented in fulfillment of the requirements for C. E. 404, (213), Structural Research.

As was originally planned, this report will serve as the first draft of a report for publication in the Welding Journal's Research Suppliment. The final paper, which is scheduled to be published in the fall 1951, will be under the authorship of Robert L. Ketter, Lynn S. Beedle and Bruce G. Johnston.

A part of the material included here-in, that of section II and III, has been taken from Progress Report L, "Interaction Curves for Columns", by Robert L. Ketter and Lynn S. Beedle.

The writer acknowledges the help and cooperation of members of the Fritz Engineering Laboratory Research staff.

Sincerely,

Robert L. Ketter
Research Ass't.

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INTRODUCTION

The study of steel columns at Lehigh University is part of a larger investigation into the behavior of welded continuous frames and their components which has as its eventual objective "to make use in analysis and design of the additional carrying capacity of statically indeterminate welded steel frames loaded beyond the elastic limit".

To accomplish this, a determination of the behavior of steel beams, columns, and continuous welded connections must first be made with development of theories to predict such behavior. Consequently, one of the objectives of this investigation is to determine the Strength of Columns Under Combined Bending and Thrust.

Unlike the two other Column Research Council sponsored projects* concerning the investigation of columns in frames, the Lehigh investigation has thus far considered columns as individual elements of a frame subjected to various known end conditions of loading.

In this report the various loading conditions considered will be referred to by the use of letter designations to describe the combination of axial thrust and end bending moment. These conditions are as follows:

Fig. 1

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* "Columns as Part of Frameworks", u nder the directorship of Dr. G. C. Kavanagh at Penn. State College

"The Buckling of Rigid Joint Structures", under the directorship of Dr. G. Winter at Cornell University.

II. THE INTERACTION CURVE CONCEPT

One means of expressing strength of an eccentrically loaded column is by use of an interaction curve.

Consider the following eccentrically loaded column.

Fig. 2

This loading can be respresented as axial loads and end bending moments acting at the ends of a pin ended member.

Fig. 3

If the eccentricities, e_1 and e_2, equal zero, a condition of axial load exists. If, on the other hand, the eccentricities are allowed to approach infinity, the member is subjected to pure bending.

Two-Dimensional Interaction Curve

Consider first the case where the eccentricities equal zero, concentric axial loading. Ifaxial load is plotted vertically against no other function, the straight-line relation of Fig. 4 is obtained. Assuming the idealized stress-strain relation Fig. 7, collapse occurs at P_y or P_E depending on L/r.

Fig. 4

Consider, next, the case where the eccentricities are allowed to approach infinity, pure bending. If the bending moment, M, is plotted horizontally, Fig. 5 is obtained.

Fig. 5

When M = M_y = $\sigma_y S$, initial yield of the extreme fiber will occur. Increasing the moment sufficiently beyond this point results in collapse as defined by the simple plastic theory as the "Plastic Hinge Value", where M = M_p = $\sigma_y Z$. The nomenclature will be found in Appendix A.

If Fig's. 4 and 5 are combined, measuring axial load as the ordinate and moment as the abscissa, there results a plot whereby all values of eccentricity can be considered.

Fig. 6

Fig. 7

The strength, or carrying capacity, of the type of member under investigation is essentially a function of the applied axial thrust and end bending moment. For zero axial load, (P = 0), the member is a beam and initial yield and ultimate collapse are determined from elastic and plastic beam theory. For the case of zero bending moment, we have a compression member whose ultimate strength is given by the Tangent Modulus Theory. It is the purpose of the next section of this report to develop equations for determining points between these two extremes as a function of a number of variables and properties of the section under consideration. The curves shown in Fig. 6 are typical of the simplest case, (L/r = 0).

Columns investigated thus far in the experimental program, were bent about the strong axis, as is customary in practice. However, the theory developed (Particularly for the collapse case) does not take into account lateral buckling; thus in a strict sense it is applicable only to columns bent about the weak axis, those which do not tend to buckle due to combined bending and twist. This is a serious limitation. The development of inelastic lateral buckling (combined bending and torsion) curves to predict behavior under these same loading conditions is considered essential. Several Column Research Council groups are working on this or related problems.

In present day engineering practice use of the concept of interaction curve equations is best illustrated by design specifications.

A.I.S.C.[8] $$\frac{f_a}{F_a} + \frac{f_b}{F_b} \geqq 1$$

A.A.S.H.O.[9] $$f_s = \frac{f_y/n}{1 + \left(0.25 + \frac{ec}{r^2}\right) B \operatorname{Cosec} \phi}$$

A.R.E.A.[10] $$p = \frac{y/f}{1 + \left(\frac{e_1 c_1}{r_1^2} + 0.25\right) \operatorname{Sec} \frac{0.875 l}{2 r_1}\sqrt{\frac{f_p}{E}} + \frac{e_2 c_2}{r_2^2} \operatorname{Sec} \frac{0.875 l}{2 r_2}\sqrt{\frac{f_p}{E}}}$$

Each of these is an interaction curve equation. In the first, A.I.S.C., moment and axial load are considered as being caused by independent conditions. In the latter two, A.A.S.H.O. and A.R.E.A., moment is considered to be caused by an initial curvature plus a known eccentricity of loading multiplied by the axial load, P. These equations will be discussed more in detail in later sections of this report.

Three-Dimensional Interaction Curve

The interaction curve previously defined may be expanded to include another important variable, length.

Fig. 8

If the z-coordinate axis is added, upon which are plotted functions of length (for illustration plot L/r as the abscissa in the (length)-axial load plane, and Ld/bt as the abscissa in the (length)-moment plane), then for any one rolled shape under a particular loading condition, the surface in space describes the carrying capacity of the column. (Since the "Ld/bt" formula was developed for uniformly-loaded, simply supported beams, it is not completely justified to make the comparison indicated. However, it serves to illustrate the idea. Test condition "c" approaches this case.)

Other Influencing Factors

In addition to lateral buckling, some of the other factors that will influence the results but which have not been considered in this paper are:

a) Residual Stress: At higher axial loads this may be a serious problem. Already in the experimental program it has been found that the initial yield value predicted by small coupon tests has not been reached when the axial load was proportionately "high".*

b) Sidesway:

c) Cross-sectional Shape: As recommended by Bleich[4], it will be necessary to develop "shape factors" for the various commercially available shapes.

d) Stress-Strain Properties: The idealized curve of Fig. 7 has been assumed.

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* Research Committee A, Materials, of the Column Research Council in their report to the annual meeting of the C.R.C., May 1951, stated that "research on the influence of residual stresses should be undertaken".

III. DERIVATION OF INTERACTION CURVE EQUATIONS*

Initial Yield Interaction Curves:

Since end moment is a more closely defined value than maximum moment which, for some loading conditions, does not occur at the end of the column; the moment applied at the end of the member has been chosen as the interaction curve abscissa. As a consequence of this choice of coordinates, a linear relationship between the axial load, P, and the end moment, M_o, will not always exist because yielding will occur at the point of maximum moment.

As a staring point, consider a very short member fo r which L/r approaches zero. Yielding will occur when

$$\sigma_y = \frac{P}{A} + \frac{M_{max}}{S} \qquad \text{---------------(1)}$$

But since L approaches zero, M_o, the moment at the end, will equal M_{max}. Therefore,

$$\sigma_y = \frac{P}{A} + \frac{M_o}{S} \qquad \text{---------------(2)}$$

Note that a linear relationship exist between P and M_o since all other quantities in the above expression are constant for any one particular cross-section.

The interaction curve for this case is plotted in Fig. 9.

Fig. 9

---

* Equations similiar to those derived herein for conditions a, c, and d are shown in Reference 11.

Next consider the case of a column of length L subjectedto the loading shown in Fig. 10a. Using graphical superposition for illustration, the moment along the column will be composed of two parts; that due to the imposed end moments, M_A and M_B, and that due to the axial load, P, multiplied by the deflection, y.

Fig. 10

If the moment diagram shwon in Fig. 10b is maintained constant while that shown in Fig. 10c is allowed to change, the tendency will be to cause the distance to the section of maximum moment from the end M_A to increase with increasing values of P. Such a progression of change is shown in Fig. 11.

Fig. 11

The value of axial load, P_2, that will just maintain a moment gradient of zero at the end of the column, x = 0, Fig. 11b, is of major importance since for any value of P less than or equal to P_2 the maximum moment will occur at the end of the column. Thus, the interaction curve for this range will be a straight line. In the remainder of this report, this value of axial load, P_2, will be noted at P_{lim}.

Loading Condition "b"

The general line of reasoning used for this case will be typical of those to be considered later.

The distinguishing feature of this type of loading is that one end is kept "fixed" (not allowed to rotate), while a moment is applied at the other end.

Consider the column with axial load and end moments as shown in Fig. 12.

Fig. 12

The general equation of the deflection curve[1] for such loading is:

$$y = -\frac{M_B}{P}\left[\frac{\sin kx}{\sin kL} - \frac{x}{L}\right] + \frac{M_A}{P}\left[\frac{\sin k(L-x)}{\sin kL} - \frac{(L-x)}{L}\right] \qquad \text{-----(3)}$$

If this is differentiated with respect to x an equation is obtained for the slope at any point along the deflected beam. By letting $x = L$ and setting the resulting slope at point $B = 0$, we can solve for M_B in terms of M_A[1].

$$\theta_B = y' = -\frac{M_B L\psi}{3EI} + \frac{M_A L\phi}{6EI} \qquad \text{-----(4)}$$

where;

$$\psi = \frac{3}{u}\left[\frac{1}{\sin 2u} - \frac{1}{2u}\right] \qquad u = \frac{kL}{2}$$

$$\phi = \frac{3}{2u}\left[\frac{1}{2u} - \frac{1}{\tan 2u}\right]$$

Since $\theta_B = 0$

$$\frac{M_B L}{3EI}\psi = \frac{M_A L}{6EI}\phi \qquad \text{or} \qquad M_B = \frac{\phi}{2\psi} M_A$$

Substituting this value in the general equati on for deflection, an equation for this particular loadin g condition is obtained.

$$y = \frac{M_A}{P}\left[\frac{\sin k(L-x)}{\sin kL} - \frac{(L-x)}{L} - \frac{\phi}{2\psi}\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right)\right]$$

Differentiating this expression for deflection, y, twice with respect to x results in the equation for curvature along the member.

$$y'' = \frac{M_A}{P}\cdot\frac{k^2}{\sin kL}\left[-\sin k(L-x) + \frac{\phi}{2\psi}\sin kx\right]$$

But curvature, ϕ, $= -\frac{M}{EI} = y''$

Therefore,

$$M = \frac{M_A}{\sin kL}\left[\sin k(L-x) - \frac{\phi}{2\psi}\sin kx\right] \qquad \text{-----(5)}$$

Since yielding occurs at the point of maximum moment, by differentiating the moment with respect to x and equating to zero, it is possible to solve for the distance to the point of maximum moment in terms of the applied axial load, P.

$$\frac{dM}{dx} = -\frac{M_A k}{\sin kL}\left[\cos k(L-x) + \frac{\phi}{2\psi}\cos kx\right] = 0$$

Therefore,

$$\cos k(L-x) = -\frac{\phi}{2\psi}\cos kx$$

or

$$X = \frac{\operatorname{Tan}^{-1}\left[\dfrac{-\frac{\phi}{2\psi} - \cos kL}{\sin kL}\right]}{k} \qquad \text{------}(6)$$

If the above equation, equation (6), is multiplied by 1/L, all quantities in the right hand side of the expression are functions of 2u where 2u = kL. The resulting plot of this equation of 2u - vs - x/L is shown in Fig. 13.

Fig. 13

To find the axial load, P_{lim}, that will be required to maintain a moment gradient of zero at the end of the column, set x = 0 in equation (6). For this to occur

$$\frac{\phi}{2\psi} = -\cos kL$$

This equation will have a solution when 2u = 2.33. Therefore,

$$2u = 2.33 = kL = L\sqrt{\frac{P}{EI}}$$

Or,

$$P_{lim} = \frac{(2.33)^2 EI}{L^2}$$

However, the Euler buckling load for one end fixed is given by the equation

$$P_{E'} \cong \frac{\pi^2 EI}{(0.7L)^2}$$

Solving this for L^2-

$$L^2 \cong \frac{\pi^2 EI}{(0.7)^2 P_{E'}}$$

Substituting in the equation for P_{lim} this value of L^2 gives,

$$P_{lim.} = 0.27\, P_{E'} \qquad \text{-----(7)}$$

Therefore, as long as P is less than or equal to 0.27 $P_{E'}$, the maximum moment will occur at the end of the column.

A typical interaction curve for this condition of loading is shown in Fig. 14. ($L/r = 112$, has been chosen for illustration).

Fig. 14

Loading Condition "d"

Consider the column with end moment and axial load as shown in Fig. 15.

Fig. 15

The equation of the elastic curve is

$$y = \frac{M_0}{P}\left[\frac{\sin k(L-x)}{\sin kL} - \frac{(L-x)}{L}\right] \qquad \text{-----(8)}$$

Differentiating this expression twice with respect to x results in the equation of curvature.

$$y'' = -\frac{M_0}{P}k^2\left[\frac{\sin k(L-x)}{\sin kL}\right] = -\frac{M}{EI}$$

Therefore,

$$M = \frac{M_0}{\sin kL}\left[\sin k(L-x)\right] \qquad \text{-----(9)}$$

To determine the distance to the section of maximum moment, differentiate M with respect to x and set equal to zero.

$$M' = \frac{M_0}{\sin kL}\left[k \cos k(L-x)\right]$$

For this to occur

$$\cos k(L - x) = 0$$

Or

$$x = L - \frac{\pi}{2K} \qquad \text{-----(10)}$$

Expressing this distance, x, in terms of the axial load, P, and the Euler buckling load, P_E ;

$$x = L - \frac{L}{2}\sqrt{\frac{P_E}{P}}$$

When $x = 0$, $P = P_{lim}$, the value of axial load causing a moment gradient of zero at the end of the column. This will occur when

$$P_{lim} = \tfrac{1}{4} P_E$$

Substituting the value of x, Equation 10, in equation 9, the general equation for moment at any section, results in an equation of maximum moment in terms of the applied end moment and the axial load.

$$M_{max} = \frac{M_0}{\sin kL}\left[\sin k\left(\frac{\pi}{2k}\right)\right] = \frac{M_0}{\sin kL} \qquad \text{-----(11)}$$

Since yielding will occur at the section of maximum moment, the initial yield interaction curve equation is

$$\sigma_y = \frac{P}{A} + \frac{M_0}{S \sin kL}$$

Or

$$M_0 = S \sin kL\left[\sigma_y - \frac{P}{A}\right] \qquad \text{-----(12)}$$

A typical initial yield interaction curve for this loading condition is shown in Fig. 16. (L/r = 112 has been selected for illustration.)

Fig. 16

Loading Condition "a"

The double curvature loading condition as shown in Fig. 17 is oftern encountered in tier buildings.

Fig. 17

This condition is identical to condition "d" with $\frac{1}{2}$L of condition "a" equaling L of condition "d". Therefore, only the resulting equation will be given. The line of reasoning used in solution of this problem is exactly the same as in the previous cases.

The equation of the deflection curve is

$$y = \frac{M_0}{P}\left[\frac{\sin kx}{\sin kL} + \frac{x}{L} + \frac{\sin k(L-x)}{\sin kL} - \frac{(L-x)}{L}\right] \qquad \text{-----(13)}$$

Therefore,

$$M = \frac{M_0}{\sin kL}\left[-\sin kx + \sin k(L-x)\right] \qquad \text{-----(14)}$$

Proceeding as before to find the distance to the section of maximum moment;

$$x = \frac{L}{2} - \frac{\pi}{2k} = \frac{L}{2} - \frac{L}{4}\sqrt{\frac{P_E''}{P}}$$

This in turn determines P_{lim}.

$$P_{lim.} = \frac{1}{4} P_E'' = P_E \qquad \text{-----(15)}$$

Substituting the distance to the section of maximum moment, x, in the general equation for moment, Equation 14, gives the desired interaction curve equation.

$$M_0 = S \sin \frac{kL}{2}\left[\sigma_y - \frac{P}{A}\right] \qquad \text{-----(16)}$$

The interaction curve, ($L/r = 112$), for this case is shown in Fig. 18.

Fig. 18

Loading Condition "c"

A loading condition with moments applied at both ends which impose a condition of single curvature on a column is the most drastic of those considered thus far. Consider the deflection curve shown in Fig. 19.

Fig. 19

It is evident that under this type of loading the maximum moment will never occur at the end of the column if there is any axial load applied what-so-ever. Therefore, there will be no straight-line portion of the interaction curve.

Repeating the same process used for the 3 preceding loading conditions - the equation of the deflection curve is -

$$y = \frac{M_0}{P \sin kL}\left[\sin kL + \sin k(L-x) - \sin kL\right]$$

Then,

$$M = \frac{k^2 M_0 EI}{P \sin kL}\left[\sin kx + \sin k(L-x)\right] \qquad \text{-----(17)}$$

Differentiating to find the section of maximum moment -

$$\frac{dM}{dx} = \frac{k^3 M_0 EI}{P \sin kL}\left[\cos kx - \cos k(L-x)\right] = 0$$

Or,

$$\cos kx = \cos k(L-x)$$

Solution will occur when $x = \frac{1}{2} L$.

Therefore,

$$M_{max} = M_0 \sec \frac{kL}{2} \qquad \text{-----(18)}$$

Substituting this into the equation for stress at a section of maximum moment will give the equation of the initial yield interaction curve.

$$M_0 = S \cos \frac{kL}{2}\left[\sigma_y - \frac{P}{A}\right] \qquad \text{-----(19)}$$

Fig. 20

"Simple Plastic Theory" Collapse Solution for L/r = 0*

Since the length of the specimen under consideration approaches zero, see Fig. 21, the following assumptions logically follow:

a. Buckling, lateral and local, will not occur.

b. Effect of end restraint will not influence analysis.

c. Bending moment caused by deflection is negligible compared with that due to initial end eccentricity.

Fig. 21

Consider one limit of the problem, in which the axial load, P, is equal to zero, the ultimate moment that could be applied to the "column" using the simple plastic theory is

$$M_p = \sigma_y Z$$

where Z is defined as the statical moment of the section.

* A part of this material is reproduced from work by Baker(2) and Roderick(3) and others.

M_p is known as the plastic hinge value. The stress distribution implied by this formula is shown in Fig. 22.

Fig. 22

If the moment imposed on the section were not sufficient to cause the full M_p value but greater than that causing M_y, the stress distribution would be as shown in Fig. 23.

Fig. 23

If:

Z = full statical moment of the section

Z_E = statical moment of part of section remaining elastic,

S_E = section modulus ofpart of section remaining elastic,

then

$$M_1 = \sigma_y \left[S_E + Z - Z_E \right]$$

where σ_y is the lower yield point stress.

The above is derived using beam theory since there is no axial load on the section. In Progress Report No. 1(12) the above expression has been compared with the results of simply-supported beam tests.

Consider not the effect of adding axial load to the applied end bending moment. (Note: Only the case of complete plasticity is investigated).

(a). Neutral Axis assumed to be in the Web,

If, (see Fig. 24)

P = Applied axial load,

y_o = Distance from center line to neutral axis,

w = web thickness,

Z_p = static moment of portion yielded by axial load, P,

t = thickness of flanges,

then it is possible to determine equations for P and M in terms of known physical dimensions of the section.

Fig. 24

The stress distribution that would be causedby such a loading condition, end moment plus axial load, is shown in Fig. 25.

Fig. 25

Therefore, from Fig. 25b

$$P = 2\sigma_y y_o w \qquad \text{-----(20)}$$

Or,

$$y_o = \frac{P}{2\sigma_y w} \qquad \text{-----(21)}$$

Also,

$$M_{pc} = \sigma_y (Z - Z_p) \text{ where } Z_p = w y_o^2 \qquad \text{-----(22)}$$

(b). Neutral Axis Assumed to be in the Flange,

Then

$$P = \sigma_y \left[w(d-2t) + 2\Delta y\, b \right] \qquad \text{-----(23)}$$

where, Δ_y = distance from the inside of the flange to the neutral axis, which has been assumed in the flange. (See Fig. 26).

Fig. 26

Then,

$$\Delta y = \frac{\frac{P}{\sigma_y} - w(d-2t)}{2b} \qquad \text{-----(24)}$$

Therefore,

$$M_{pc} = b\sigma_y (t-\Delta y)(d-t+\Delta y) \qquad \text{-----(25)}$$

A typical interaction curve for this condition is shown in Fig. 27.

Fig. 27

It should be emphasized that this solution is only applicable when the moment is a maximum at the end of the column. Tests of columns bent about the strong axis show that a "lateral buckling" type of failure takes place when the axial load is of such magnitude that the moment is not a maximum at the column end. Thus, the value of extending the analysis to include such a case appears academic unless the column is bent about its "weak" axis.

Ježek Buckling Solution for condition "c"

Loading condition "c" is similar to the eccentrically loaded column with equal eccentricities at each end.

F. Bleich [4] lists procedures of analysis for this condition based on the work of Ježek.

Using the same notation as Bleich, where σ_{co} = "the critical average stress of the centrally loaded column derived from an actual column curve", and σ_c = "the critical average stress of the eccentrically loaded column".

$$\beta = \frac{\sigma_{co}}{\sigma_c}$$

Multiplying the right hand side of the above equation by A/A,

$$\beta = \frac{A\sigma_{co}}{A\sigma_c} = \frac{P_{cr}}{P}$$

Approximate equations have been recommended for Structural Steel with a lower yield point of 40 ksi. These are:

For L/r = 20 to 75, $\beta = (1 + \frac{\chi}{2} + \frac{\sqrt{\chi}}{9000}\left(\frac{L}{r}\right)^2$ -----(26)

For L/r = 95 to 200, $\beta = 1 + \frac{10600}{\left(\frac{L}{r}\right)^2}\sqrt[3]{\chi^2}$ -----(27)

where χ = eccentricity ratio

$$\chi = \frac{Mr^2}{Pc} = \frac{er^2}{c}$$

Consider the range of L/r from 20 to 75. Introducing a shape factor, of 1.3, as tentatively recommended by Bleich[4]*,

$$\beta = \left(1 + \frac{1.3\chi}{2}\right) + \frac{\sqrt{1.3\chi}}{9000}\left(\frac{L}{r}\right)^2$$

---

* This "shape factor" is not the value $f = M_p/M_y$ as used in plastic theory (see nomenclature), but is a factor to be applied to the term χ dependent on the cross-sectional form and flexure axis.

Solving this equation for M will give the following interaction formula.

$$M = \frac{Pr^2}{1.3c}\left[\frac{-\left(\frac{L}{r}\right)^2}{9000} \pm \sqrt{\frac{\left(\frac{L}{r}\right)^4}{(9000)^2} - 2\left(1 - \frac{P_{cr}}{P}\right)}\ \right]^2 \qquad \text{-----(28)}$$

The deviation from an exact solution of equation (28) becomes appreciable for small values of χ, but decreases rapidly as M increases.

A resulting curve of equation (28) for one particular column, (L/r = 56, 8WF31) is shown in Fig. 28.

Fig. 28

Discussion of Interaction Curves

The relationship between the "interaction formula" and the interaction curve may be seen from Fig. 29. Here stress due to bending -vs- stress due to axial load -vs- length of column for a particular cross-section, (4WF13), have been plotted. Loading condition "d" has been chosen for illustration.

If F_a and F_b are the same values as in the AISC formula, where these determine the allowable loads under axial load alone and bending alone respectively, the yield point load or moment can be expressed as a function of these values, (i.e. $n_2F_a = P_y/A = \sigma_y$ and $n_1F_b = M_y/S = \sigma_y$).

The surface efghij formed by the straight lines ef, gh, and hi, and the curve ije is a plane, assuming an idealized stress strain diagram (shown in Fig. 7). For any point on this surface, carrying capacity is not a function of length of the column.

Fig. 29

Initial yield interaction curves have been developed for a number of standard loading conditions. The influence of length on each condition is shown by Fig. 30, the tendency being that for a certain moment, the axial load at which yielding will occur decreases as the slenderness ratio increases. However, for each loading condition, except "c" (single curvature), there is a range of axial load which is theoretically unaffected by slenderness ratio. This range decreases with increasing slenderness ratio. (This range corresponds to that range defined by the plane efghij in Fig. 29.)

Fig. 30

In comparing the strength, carrying capacity, for the various conditions, (see Fig. 31), it is evident that the severity of loading condition increases in the order "a", "b", "d" and "c". The effect of restraining moment at one end is quite pronounced. (Compare "a" and "b" with "c" and "d".)

Fig. 31

It is well to note that even though in Fig. 29 a curve for M_{cr} is included, the derivations in this report do not take into account lateral buckling, (bending ≠ twist). Further analytical work is needed to include this important condition, especially in the inelastic range.

A method for determining the collapse interaction curve using the simple plastic theory for $L/r = 0$ has been reviewed. It does not take into account the influence of slenderness ratio. Also included, is an approximate buckling solution derived by Ježek. (It is apparent that the "shape factor" as suggested by Bleich is not a constant but varies for each member.)

Collapse solutions for eccentrically loaded columns based on a more rigorous plastic theory, such as along the lines advocated by Ježek and Chwalla, due to complexity have not been included in this report.[6]

Non-dimensional interaction curves are of value since they are independent of particular magnitudes of stress, load or moment. Such curves and their development have been discussed by Shanley[7] and others. One such type of curve, that where the coordinate axes have been chosen as P/P_y and M/M_y respectively, will be used later in this report in the presentation of test results.

IV COMPARISON OF INITIAL YIELD INTERACTION CURVES WITH DESIGN SPECIFICATIONS

Present design specifications for the type of structures encountered in Civil Engineering practice are in general based on initial yield as the criterion of failure. Therefore, it should be possible to show that for certain assumptions these design equations reduce identically to the initial yield interaction curve equations derived in Section III of this report.

A.A.S.H.O. Column Specifications(9)

$$\frac{P}{A} = \frac{f_y/n}{1 + \left(0.25 + \frac{e_g c}{r^2}\right) B \operatorname{Cosec} \phi}$$

where n = factor of safety which takes account of variance of mechanical properties, etc.,

$$B = \sqrt{\alpha^2 - 2\alpha \operatorname{Cos} \phi + 1} \quad ; \quad \alpha_a = -1, \ \alpha_c = +1, \ \alpha_d = 0$$

and

$$\phi = \frac{L}{r}\sqrt{\frac{Pn}{AE}}$$

Assuming that the initial end moment, M_o, for the column under investigation is caused by a known eccentricity, e_g, and an initial curvature in the plane of the known eccentricity, 0.25, -then ,

$$M_0 = P\left(\frac{0.25 r^2}{c} + e_g\right)$$

Substituting this in the A.A.S.H.O. formula above, noting that $\phi = kL\sqrt{n}$, results in the following equation.

$$\frac{f_y}{\eta} = \frac{P}{A} + \frac{M_o \beta}{S} \operatorname{Cosec} kL\sqrt{\eta}$$

Using a safety factor equal to one, $\eta = 1$, this equation reduces to

$$M_o = \frac{S}{\beta} \operatorname{Sin} kL \left[\sigma_y - \frac{P}{A} \right]$$

For the various loading conditions defined by B, this equation is identically equal to those previously derived.

<u>A.R.E.A. Specification for Structural Steel Compression Members</u>(10)

$$\frac{P}{A} = \frac{y/f}{1 + \left(\frac{e_1 c_1}{r_1^2} + 0.25\right) \operatorname{Sec} \frac{L}{2r_1}\sqrt{\frac{fP}{AE}} + \frac{e_2 c_2}{r_2^2} \operatorname{Sec} \frac{L}{2r_2}\sqrt{\frac{fP}{AE}}}$$

where f = factor of safety, (in A.A.S.H.O. Spec's.).

The major difference between this specification and the A.A.S.H.O. is that in this specification bending is assumed about both axes of the section. Initial curvature is therefore assumed to occur in the weak direction.

Making the assumption that bending in the plane perpendicular to the web is caused by a known eccentricity, e_1, and an initial curvature, 0.25, and that bending in the plane of the web is caused only by a known eccentricity, e_2, the equation for moment in each of these planes is

$$M_{o_1} = P\left(\frac{0.25\, r_1^2}{c_1} + e_1\right)$$

$$M_{o_2} = P\left(e_2\right)$$

Therefore,

$$\frac{y}{f} = \frac{P}{A} + \frac{M_{o_1}}{S_1} \operatorname{Sec} \frac{kL}{2}\sqrt{f} + \frac{M_{o_2}}{S_2} \operatorname{Sec} \frac{kL}{2}\sqrt{f}$$

Consider the case where the factor of safety is assumed equal to one, f = 1. Then the general equation reduces to

$$\sigma_y = \frac{P}{A} + \frac{M_{o_1}}{S_1} \operatorname{Sec} \frac{kL}{2} + \frac{M_{o_2}}{S_2} \operatorname{Sec} \frac{kL}{2}$$

which is identical to our condition "c", (single curvature), for the case of bending in both axes. (Use super-position of the stresses caused by the two moments and the axial load.)

A.I.S.C. Interaction Formula(9)

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \geqq 1 \, ,$$

$$f_a = \frac{P}{A}$$

$$f_b = \frac{Mc}{I}$$

$$F_a = 17000 - 0.485\left(\frac{L}{r}\right)^2$$

$$F_b = \frac{12000000}{\frac{ld}{bt}} \qquad \text{for } \frac{ld}{bt} > 600$$

$$= 20000 \qquad \text{for } \frac{ld}{bt} < 600$$

Since the Ld/bt formula was derivedfor a uniformly loaded, simply supported beam, it is impossible to directly compare it with the equations in section III.

V EXPERIMENTAL INVESTIGATION

Description of Test Apparatus

In order that end bending moment could be applied independently of axial thrust, a method such as that shown in Fig. 32 has been used in the experimental part of the investigation.

Fig. 32

End moments are accomplished by exerting forces at the ends of lever arms rigidly attached to the ends of the test specimens. These forces, applied by hydraulic jacks, are accurately measured by aluminum tube dynamometers. The axial thrust is applied by an 800,000# Riehle testing machine. Details of the testset-up and testing proceedure are given in Reference 13.

Column Tests Completed

Table I presents a summary of the test conditions of the various columns tested and reported herein. Two geometrically similar sections were chosen for investigation, 8WF31 and 4WF13. Using these sections with lengths of 8 ft., 12 ft., and 16 ft./ has made it possible to test columns in a range of L/r extending from L/r = 28 to 112.

Table I

TESTS COMPLETED

Test Number	Section	Length*	Loading Condition	Test Condition	
				P/P_y	M/M_y
Pilot	4WF13	16'	e, d, b	0.26,0.5	-
1	8WF31	6'	d	0.13	-
2	8WF40	6'	d, b	0.1,0.14,0.15	-
3	8WF31	16'	b	0.5	-
4	8WF31	16'	b	0.12	-
5	8WF31	16'	b	0.8	-
6	4WF13	16'	b	0.26	-
7	4WF13	16'	b	0.26	-
8	8WF31	16'	c	-	0.13
9	4WF13	16'	b	0.10	-
10	4WF13	16'	b	0.49	-
11	8WF31	16'	c	-	0
12	8WF31	16'	c	0.12	-
13	8WF31	16'	d	0.12	-
14	8WF31	16'	a	0.12	-
15	8WF31	12'	c	-	0
16	8WF31	12'	c	0.12	-

* Add $11\frac{1}{2}$" to obtain exact distance between knife edges.

Since end moment is applied independent of axial thrust, it is possible to test under several different test conditions. These are: a) holding the end moment constant while increasing the axial thrust, b) holding the axial load constant while allowing the end moment to increase, and c) increase both in any desired ratio. Because of this flexibility, any condition from a pin end axially loaded column to one of any desired restraint can be simulated.

Test Results

The results of the tests listed above are presented in Fig's. 34 thru 41 . Here the experimentally determined values of strength are plotted on interaction curves.

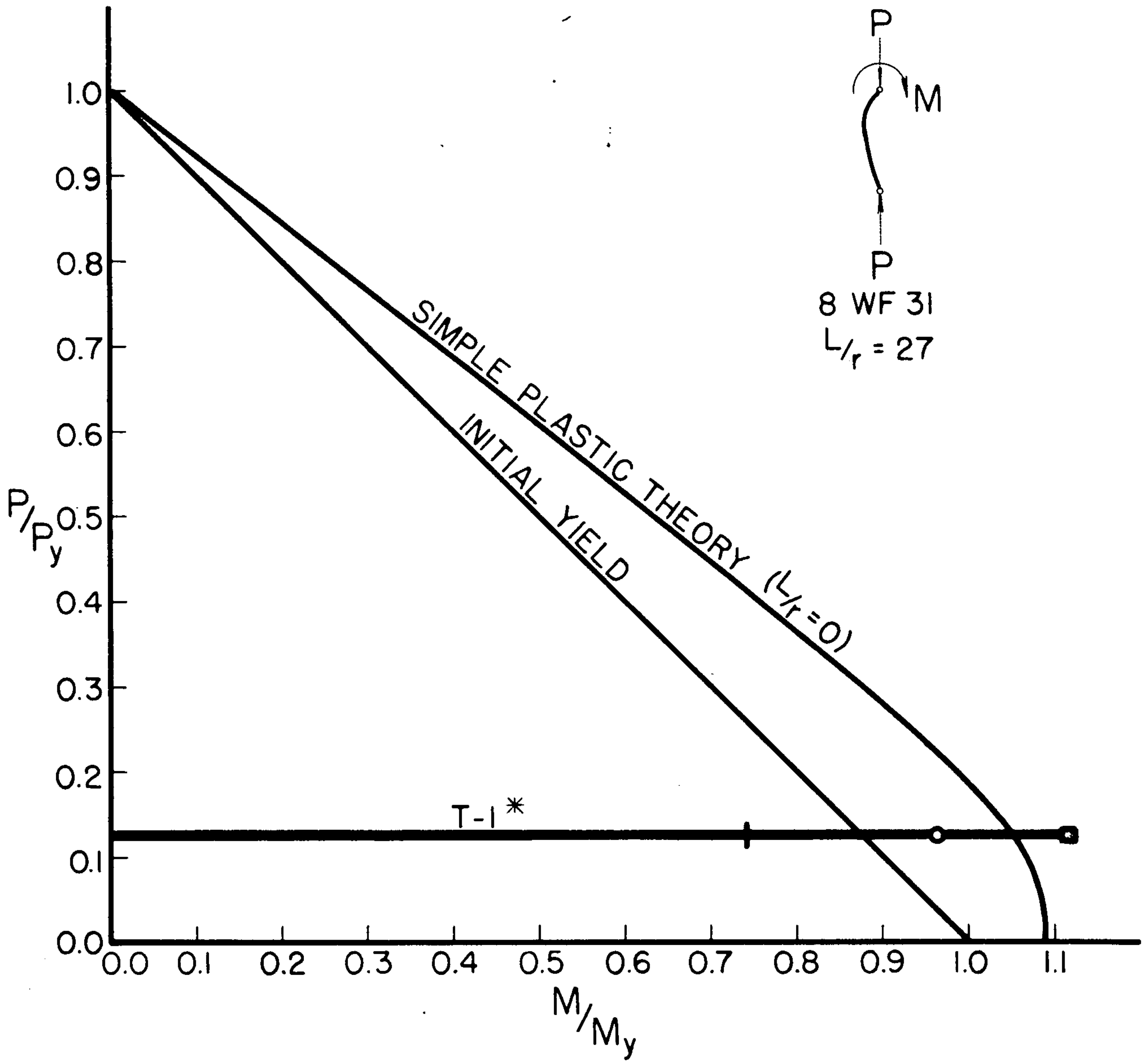
Three values of strength are presented.

1. 1st. yield line
2. yield strength
3. collapse

These conditions are graphically defined by the moment-rotation curve of Fig. 33.

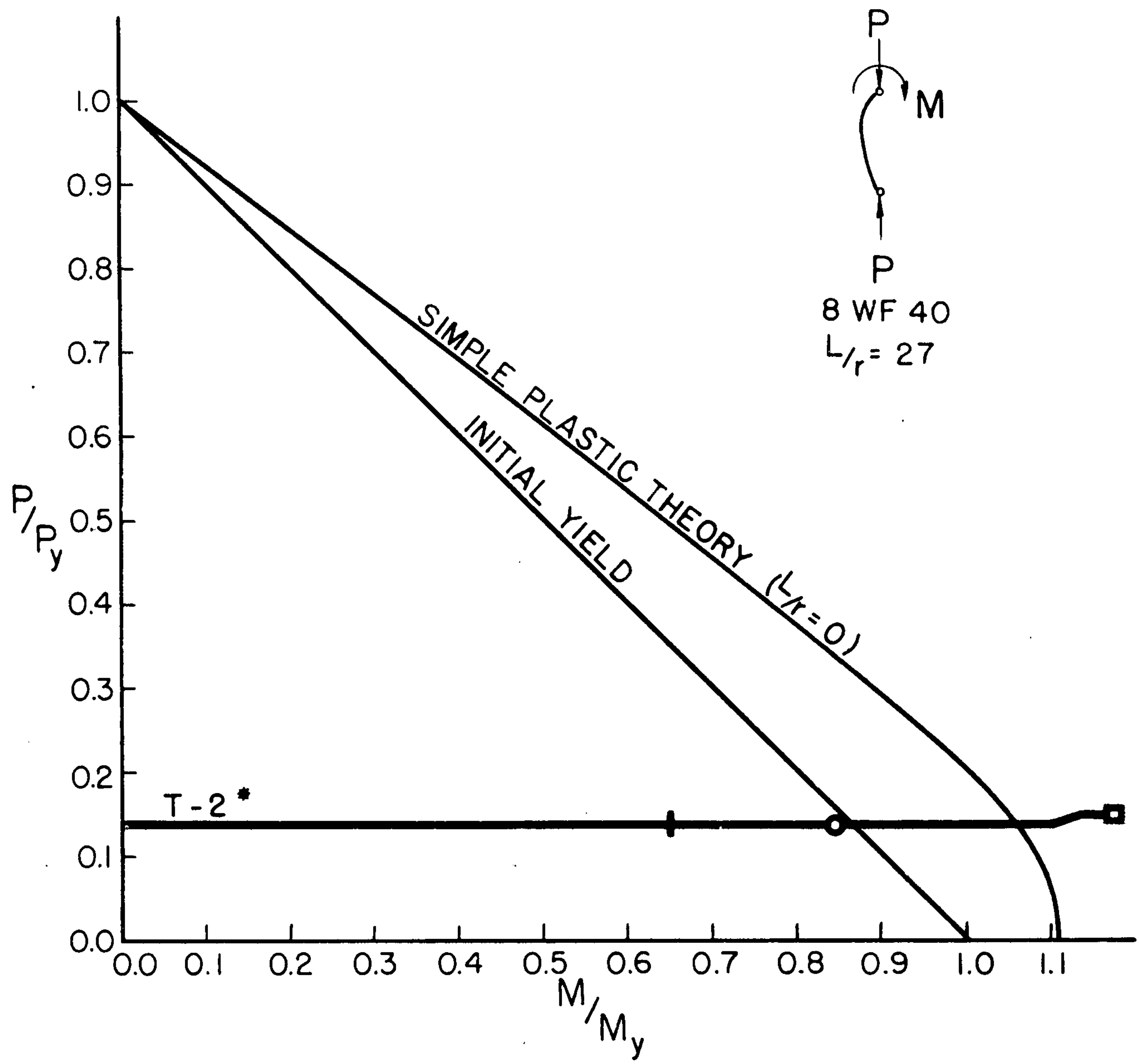
Fig. 33.

Double weight lines are shown on the interaction curves to indicate the path chosen for attaining the desired position on the interaction curve. At the point at which the 1st yield line occurred, a dash perpendicular to the direction of testing is shown. At the point where the yield strength was attained, a circle is shown. For the case of collapse, a square has been used.



* Length of Column T-1 was 6'-0".

Fig. 34



* Length of Column T-2 was 6'-0"

Fig. 35

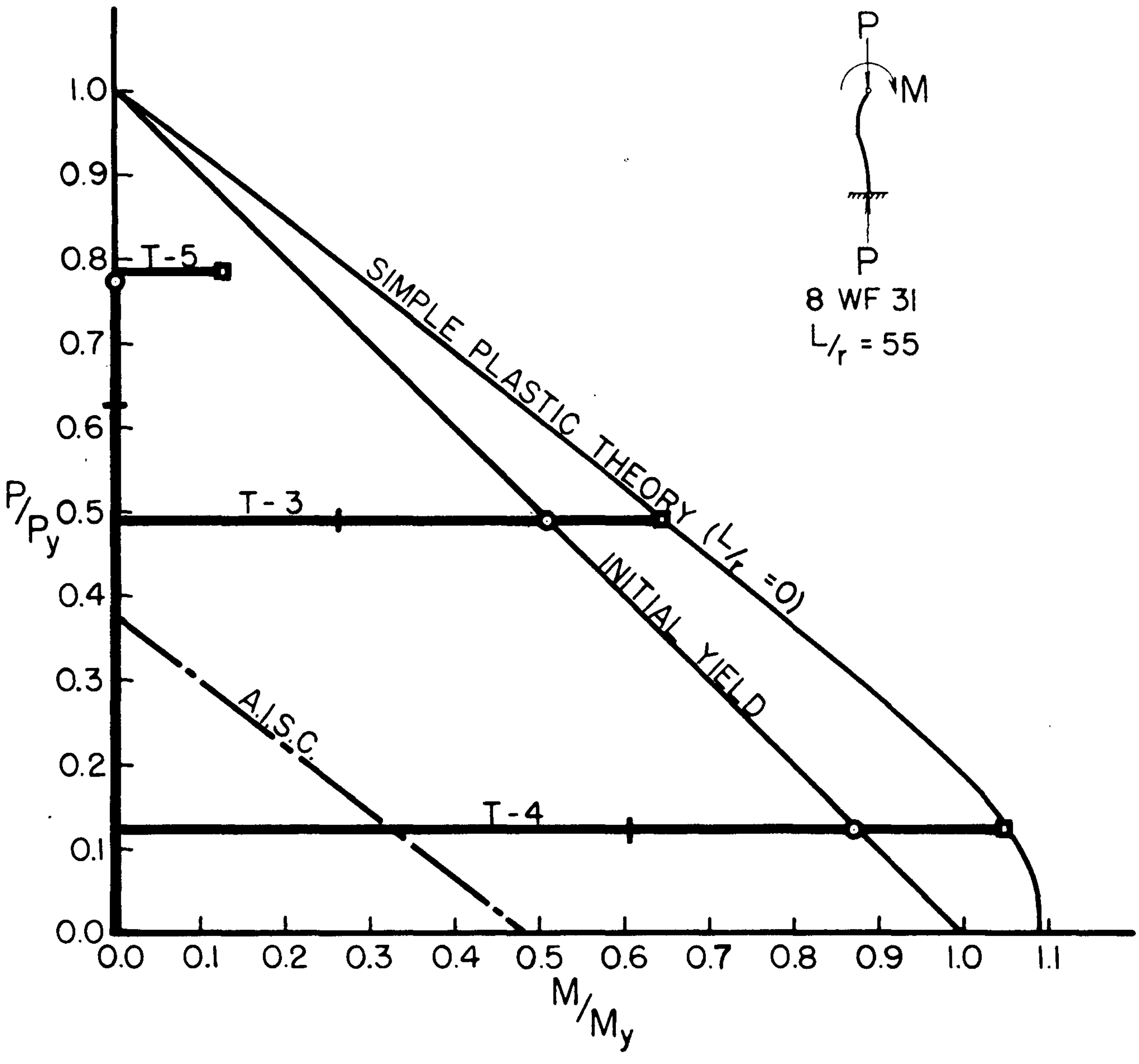
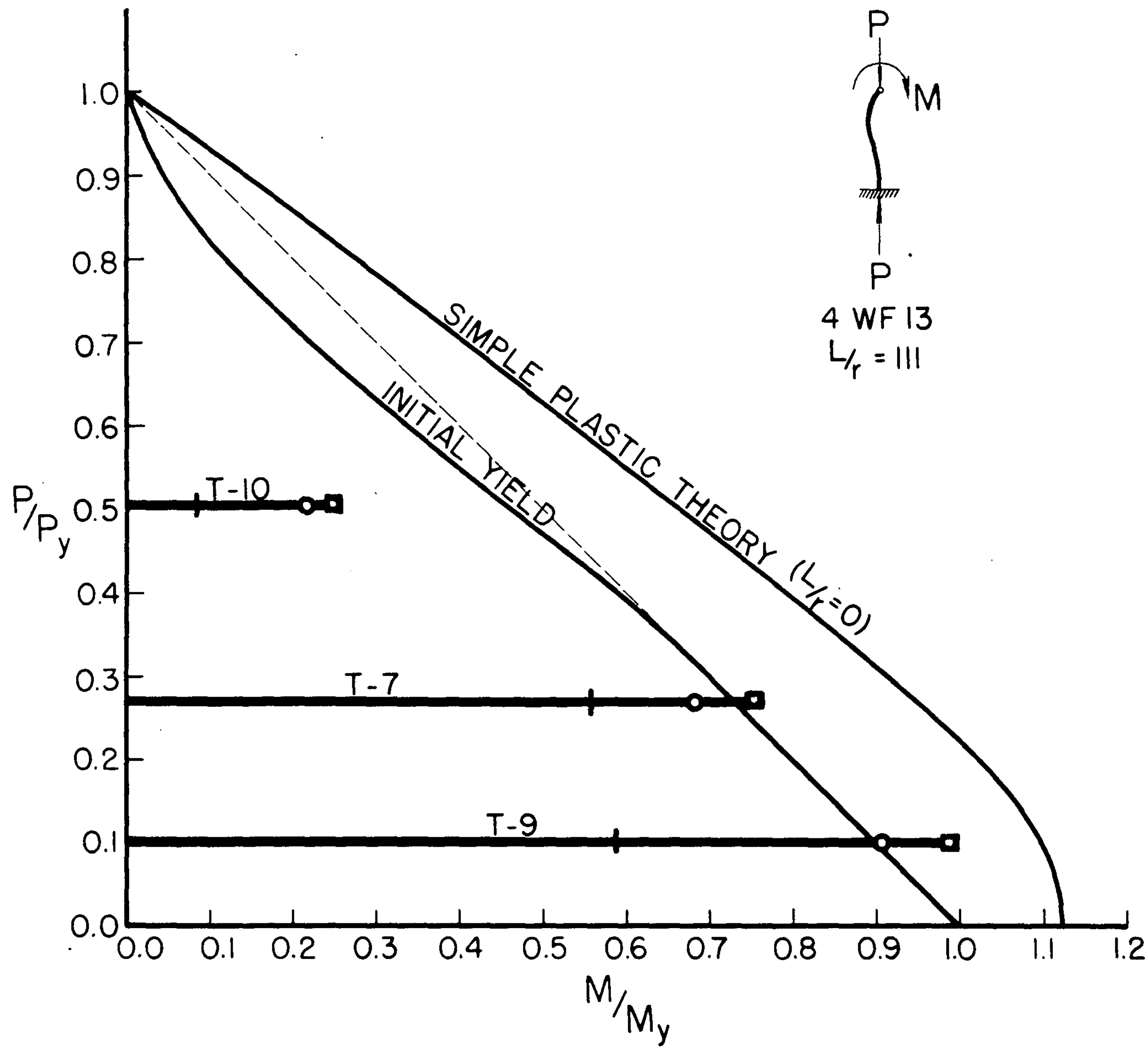

P
M
P
8 WF 31
L/r = 55
SIMPLE PLASTIC THEORY (L/r = 0)
INITIAL YIELD
A.I.S.C.
T-5
T-3
T-4
P/Py
M/My
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

Fig. 36



P
M
P
4 WF 13
L/r = 111
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
P/Py
T-10
T-7
T-9
SIMPLE PLASTIC THEORY (L/r=0)
INITIAL YIELD
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
M/My

Fig. 37

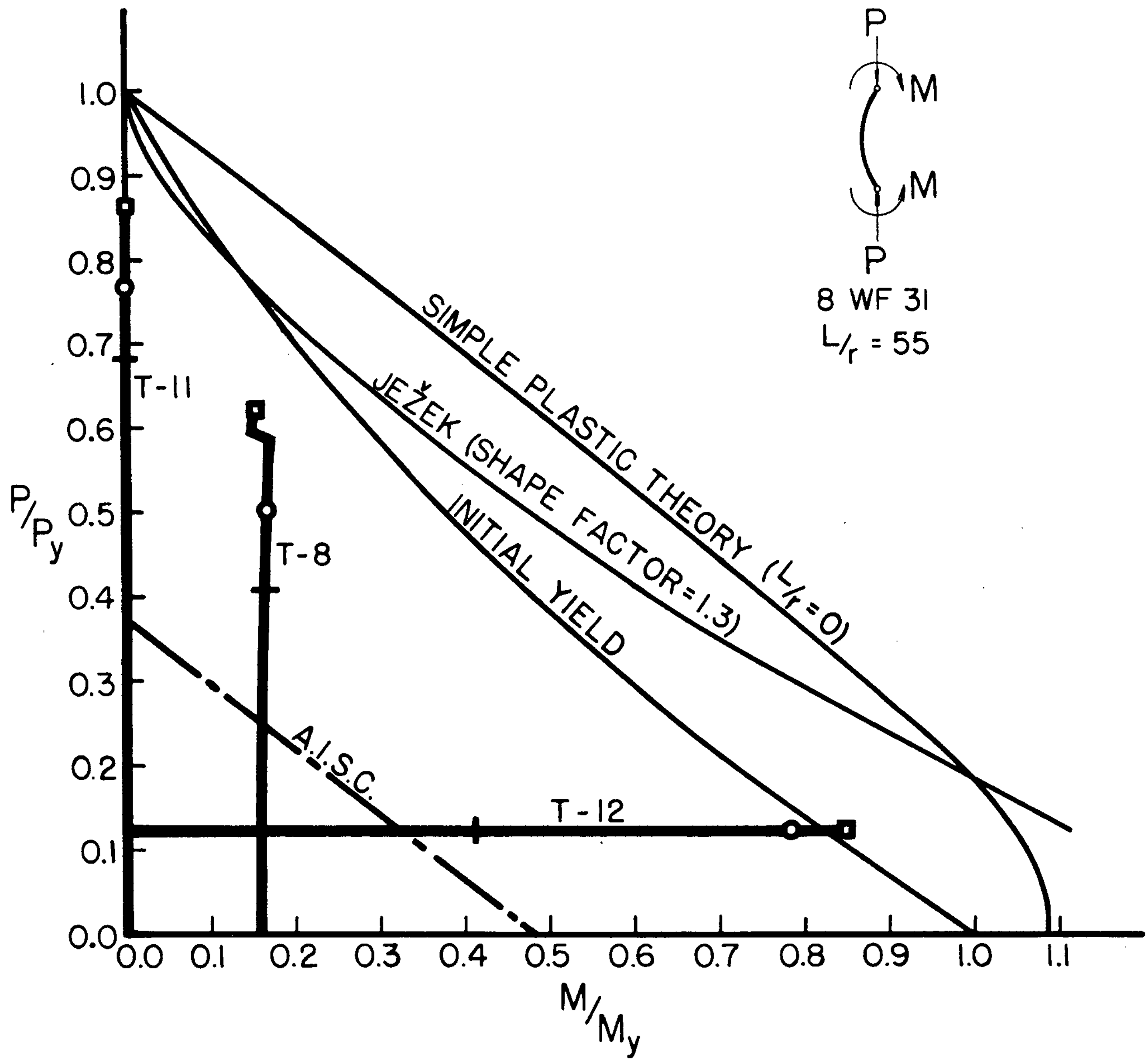

P
M
M
P
8 WF 31
L/r = 55
SIMPLE PLASTIC THEORY (L/r=0)
JEŽEK (SHAPE FACTOR=1.3)
INITIAL YIELD
A.I.S.C.
T-11
T-8
T-12
P/Py
M/My
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

Fig. 38

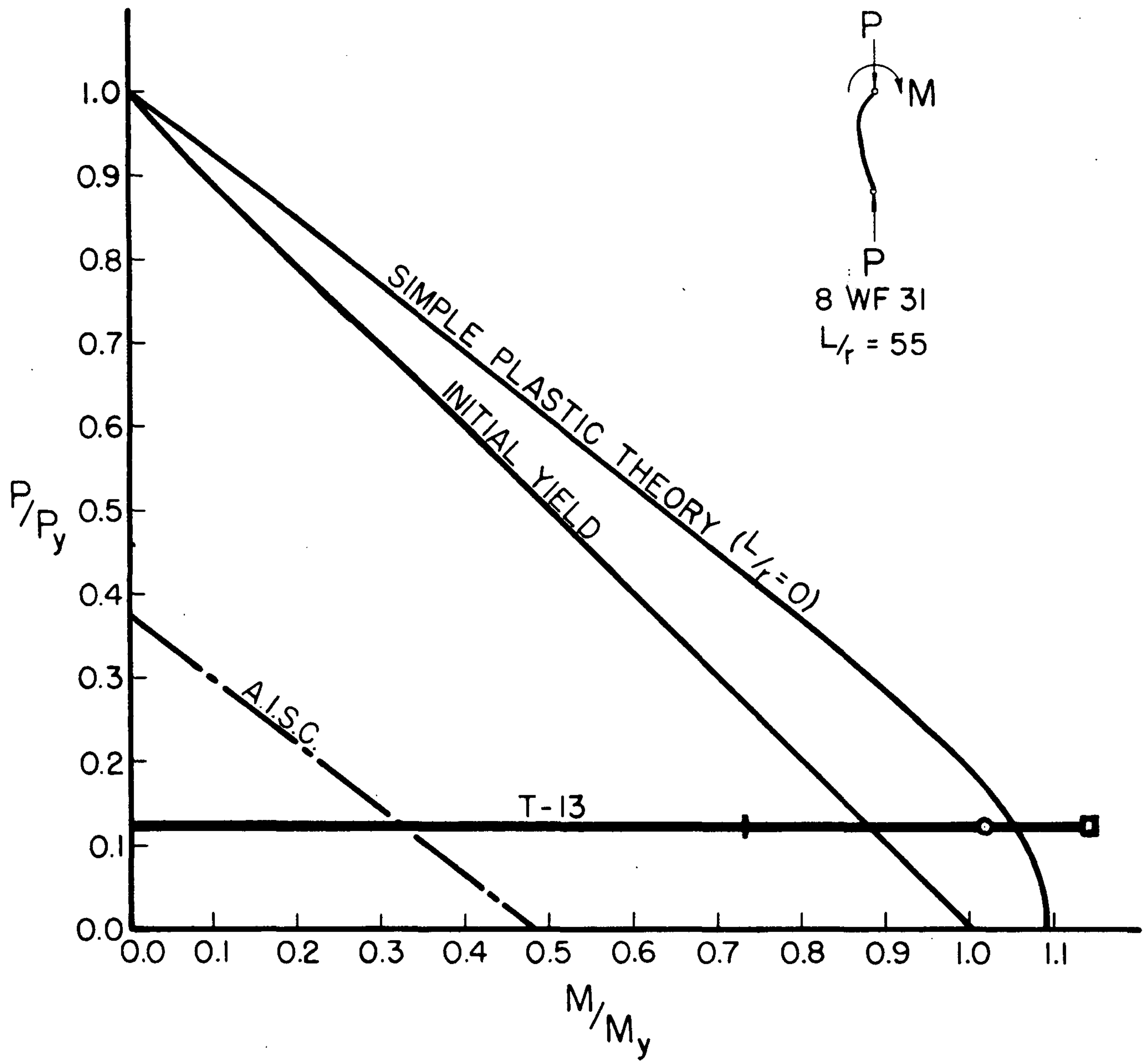

P
M
P
8 WF 31
L/r = 55
SIMPLE PLASTIC THEORY (L/r = 0)
INITIAL YIELD
A.I.S.C.
T-13
P/Py
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1
M/My

Fig. 39

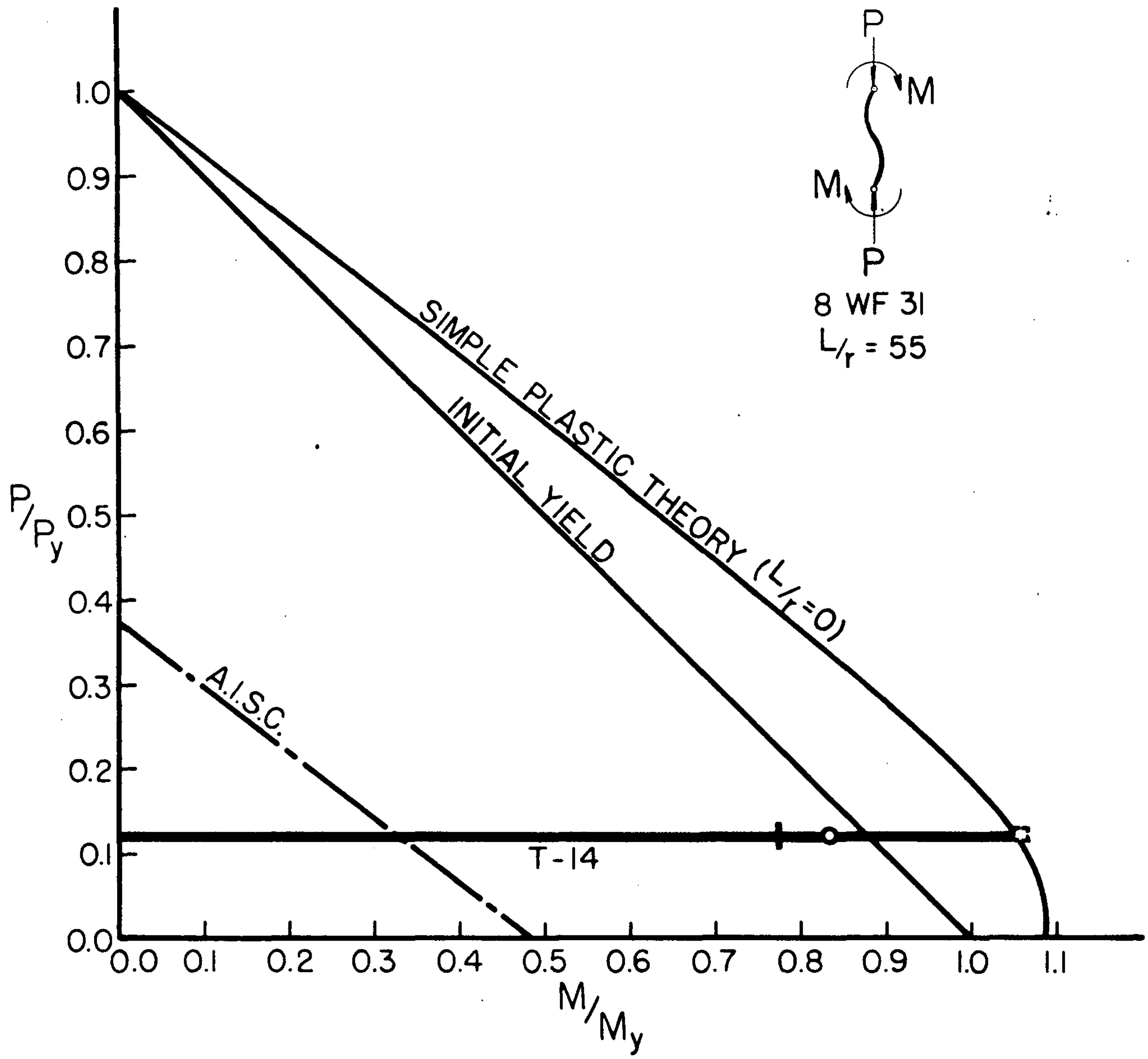

P
M
M
P
8 WF 31
L/r = 55
SIMPLE PLASTIC THEORY (L/r = 0)
INITIAL YIELD
A.I.S.C.
T-14
P/Py
M/My
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1

Fig. 40

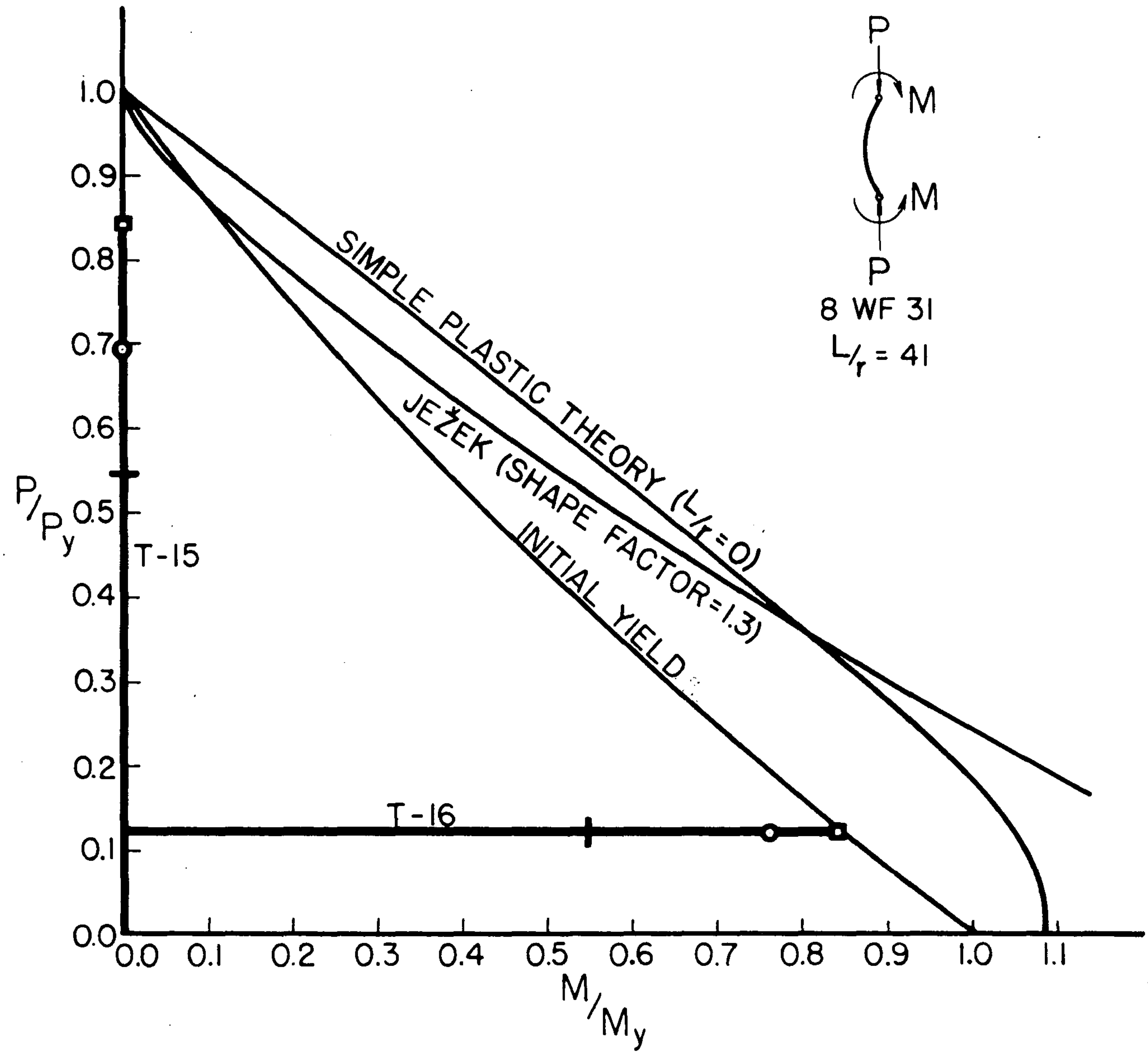

P
M
M
P
8 WF 31
L/r = 41
SIMPLE PLASTIC THEORY (L/r=0)
JEŽEK (SHAPE FACTOR=1.3)
INITIAL YIELD
T-15
T-16
P/Py
M/My
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

Fig. 41

VI DISCUSSION OF TEST RESULTS AND SUMMARY

Two general typesof failure, lateral buckling and local buckling, were observed in the present investigation. Examples of these are shown in Figs. 42 and 43 .

A summary of the test results is shown in Table II. Here loading condition, P/P_y, L/r and type of failure for each specimen are listed.

Except for short columns with relatively low axial loads tested under conditions a, b, and d, failures have been of the lateral buckling type. Tests T-4, T-13 and T-14 are the exceptions. Test T-3 failed due to a combination of both lateral and local buckling.

Loading condition c, single curvature, has in all cases regardless of L/r or P/P_y failed due to a lateral buckling type of failure. However, the carrying capacity has been appreciably reduced when the value of P/Py was relatively high. This reduction is in part due to lateral buckling, but it is also due to the residual stress in the member. Present knowledge indicates that a residual stress level of approximately 10-15 ksi ispresent in those rolled sections tested. This condition of residual stress would tend to aggravate the already present tendency toward lateral buckling thereby causing a further reduction in strength.

In all cases where local buckling occured, the collapse value as predicted by the simple plastic theory for $L/r =$ 0 was reached. Since local buckling occured only when the maximum moment occured at the end of the member for relatively low values of P/P_y and L/r, it is possible to postulate that there is a certain range where the simple plastic theory will give collapse values sufficiently close for design. This range decreases with increasing L/r. A similar condition has been observed by Baker in England on small model tests of columns tested under conditions similar to those reported herein as conditions a, b and d.

In some cases, (T-13, T-1 and T-2), collapse has exceeded that value predicted by the simple plastic theory for $L/r = 0$. This is attributed to strain hardening of the steel. Each of these tests when carried to collapse failed due to local buckling. Test T-14 would have also exceeded the predicted value but was stopped after having exceeded this value slightly because of the extreme difficulty of providing enough lateral force at the ends of the specimen to counteract the high shears caused by a condition "a" type of loading.

Since local buckling is dependent on the length of the yielded zone of the compression flange, and since the length of this zone is a function of the loading condition and length of the column; a short column under a steep moment gradient will have a small yielded zone therefore less tendency to local buckle. This would result in a portion of the member strain hardening, thus increasing the strength beyond that value predicted by the simple plastic theory. Therefore, short columns tested under loading condition "a" would be the first expected to exceed the predicted value. The next in line of probability would be condition "b". Next, condition "d". Since for loading condition "c", local buckling will not occur, except for extremely short members, experimental collapse will never exceed that value predicted by the simple plastic theory.

It is interesting to note that when the maximum moment occurs at the end of the member, at least the predicted initial yield value will be reached regardless of the mode of failure, (see T-7, T-9, and T-10; Fig. 37). For the condition of single curvature, the column will not develop its yield strength unless the axial load is relatively low, (see Figs. 38 and 41). For the other loading conditions as noted previously, the column will or will not develop the predicted strength depending on L/r and P.

TABLE II
SUMMARY OF TEST RESULTS

Test Number	Section	L/r	Failure Loading Condition	P/P_y	M/M_y	Type of Failure	Predicted Yield Reached	Predicted Collapse Reached L/r.0
Pilot	4WF13		b	0.26	--		+	-
1	8WF31	20.6	d	0.13	--	Plastic Hinge Developed**	+	+
2	8WF40		b	-	--	Plastic Hinge Developed**	+	+
3	8WF31	55.4	b	0.50	--	Local Buckling of Flg. & Lateral "	+	0
4	8WF31	55.2	b	0.12	--	Local Buckling of Flg.	+	0
5	8WF31		b	0.80	--	Lateral Buckling	-	-
6	4WF13	111.7	b	0.26	--	Lateral Buckling	-	-
7	4WF13	110.9	b	0.26	--	Lateral Buckling	+	-
8	8WF31	55.1	c	-	0.13	Lateral Buckling	-	-
9	4WF13	111.0	b	0.10	--	Lateral Buckling	+	-
10	4WF13	111.0	b	0.49	--	Lateral Buckling	-	-
11	8WF13	55.2	c	--	0	Lateral Buckling	-	-
12	8WF31	55.2	c	0.12	--	Lateral Buckling	+	-
13	8WF31	54.9	d	0.12	--	Local Buckling of Flg.	+	+
14	8WF31	55.2	a	0.12	--	Local Buckling of Flg.	+	0
15	8WF31	41.3	c	-	0	Lateral Buckling	-	-
16	8WF31	41.2	c	0.12	--	Lateral Buckling	0	-

** If these tests had been carried further, local buckling would have occurred.
+ Denotes exceeding value,
- Denotes not reaching value,
0 Just did reach.

SUMMARY

1. Initial yield interaction curves have been developed for a number of standard loading conditions.
2. A method for determining the collapse interaction curve, excluding the influence of L/r, has been reviewed. Reference has been made to more detailed investigations using a more rigorous theory of plasticity.
3. Derivations included in this report do not take into account lateral buckling, (bending + twist). Further analytical work is needed to include this very important condition especially in the inelastic range.
4. Two general types of failure have been observed; lateral buckling and local buckling. When local buckling occurs at least the collapse value as predicted by the simple plastic theory will be reached. When lateral buckling occurs the column will or will not develop the predicted collapse strength depending on the condition of loading, L/r and P.
5. The severity of loading conditions on strength increases in the order "a", "b", "d" and "c ". Likewise the tendency toward lateral buckling as the mode of failure increases from "a" to "c", with "c" always failing due to combined bending and twist.

6. When the maximum moment occurs away from the end of the member, a more drastic condition is imposed on the column.

7. Columns loaded such as to cause single curvature, condition "c", will not even carry the predicted yield value unless the axial load is relatively low. This reduction in strength is due to residual stresses and lateral buckling.

8. When the maximum moment occurs at the end of the column, the predicted initial yield value will at least be developed, regardless of type of failure.

9. From the experimental program it has been observed that at relatively high axial loads, residual stress appears to have a pronounced effect on carrying capacity. A preliminary investigation to determine the effect of residual stress in the 8WF31 section indicates a reduction in carrying capacity at collapse of an axially loaded member, (L/r approaching zero), of approximately 8-10% due to residual stress.

VII Acknowledgements

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This program is being carried out in the Fritz Engineering Laboratory of which Prof. Wm. J. Eney is Director.

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Appendix A

Definitions, Nomenclature and Terminology

A = area of section

b = width of section

c = distance from N.A. to extreme fiber

d = depth of section

E = Young's Modulus

f_a = actual axial unit stress at a particular section

f_b = actual bending unit stress at a particular section

F_a = axial unit stress that would be permitted if axial stress only existed

F_b = bending unit stress that would be permitted if bending stress only existed

I = moment of inertia

$K = \sqrt{P/EI}$

L = total length of the member

L/r = slenderness ratio

Ld/bt = ratio governing allowable compressive stress in flanges as specified by the AISC manual.

M = moment at any section along column

M_y = moment at which yield point is reached in flexure

M_p = "Hinge value"; full plastic moment; the ultimate moment that can be reached at a section according to the simple plastic theory. M_{ult} in Timoshenko. Collapse moment for a simple beam. $M_p = \sigma_y Z$.

M_A = moment applied at end A

M_B = moment applied at end B

M_O = moment applied at end of column

M_{pc} = collapse moment for a beam-column at a particular section. The ultimate moment or collapse load of a column as modified by compression load

M_{yc} = same as M_y except modified for compression load

P = an applied load

P_{cr} = useful column load. A load used as the "maximum column load" in design procedures. This might by P_t, P_E, P_{ult}.

P_y = the load at which yield point stress is reached while under pure axial load.

P_E = Euler buckling load for a condition where both ends of the column are pinned.

$$= \frac{\pi^2 EI}{L^2}$$

P_E' = Euler buckling load for a condition of one end being pinned, the other being fixed.

$$= \frac{\pi^2 EI}{(0.7L)^2}$$

P_E'' = Euler buckling load for a condition of both ends being pinned but subjected to a loading causing double curvature.

$$= \frac{\pi^2 EI}{(0.5L)^2}$$

r = radius of gyration

S_E = section modulus of part of section remaining elastic

S = section Modulus

$$= \frac{I}{c}$$

t = flange thickness

u = $\frac{KL}{2}$

w = web thickness

x = distance measured along column

y = deflection

y' = first derivative of y with respect to x

= slope

y'' = second derivative of y with respect to x

= curvature

y_o = distance from neutral axis to centroid of section

Z = plastic Modulus

= static moment of the entire cross-section

$= \int_{y_1}^{y_2} y\,dA$

Z_p = statical moment of part yielded by axial load, P

Z_T = full statical moment of the section

Z_E = statical moment of part of section remaining elastic

σ = stress

σ_y = yield stress

σ_{cr} = critical stress

ϵ = strain

$$\phi = \frac{3}{2u}\left[\frac{1}{2u} - \frac{1}{\tan 2u}\right]$$

$$\psi = \frac{3}{u}\left[\frac{1}{\sin 2u} - \frac{1}{2u}\right]$$

Δ = Core radius

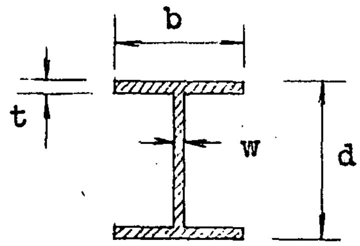
$= \frac{r^2}{c}$

χ = eccentricity ratio

$= \frac{M\Delta}{P} = \frac{Mr^2}{Pc}$

Appendix B

AVERAGE SECTION PROPERTIES



	d	b	w	t	A	I_x	S_x	r_x
8 WF 31 (Average of 11 specimens)								
Measured	8.069	8.032	0.299	0.427	9.177	111.356	27.60	3.483
Handbook	8.00	8.000	0.288	0.433	9.12	109.7	27.4	3.47
% Var.	+0.86%	+0.40%	+3.82%	-1.39%	+0.63%	+1.51%	+0.73%	+0.39%
4 WF 13 (Average of 4 Specimens)								
Measured	4.139	4.140	0.258	0.337	3.759	11.215	5.419	1.727
Handbook	4.16	4.06	0.28	0.345	3.82	11.3	5.45	1.72
% Var.	-5.05%	+4.93%	-7.86%	-2.32%	-1.59%	-0.75%	-0.57%	+0.41%
8 WF 40 (One Specimen)								
Measured	8.300	8.070	0.371	0.550	11.691	146.82	35.379	3.544
Handbook	8.25	8.077	0.365	0.558	11.76	146.3	35.5	3.53
% Variat.	+0.61%	-0.08%	+1.64%	-1.43%	-0.59%	+0.36%	-0.34%	+0.40%

FIGURES

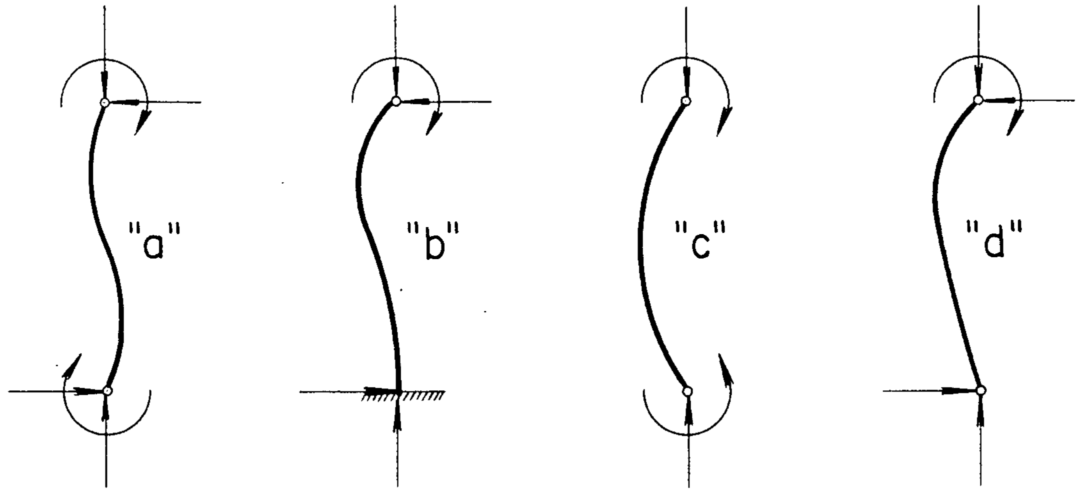


Fig. 1

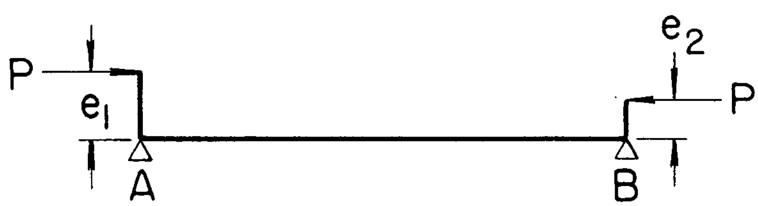


Fig. 2

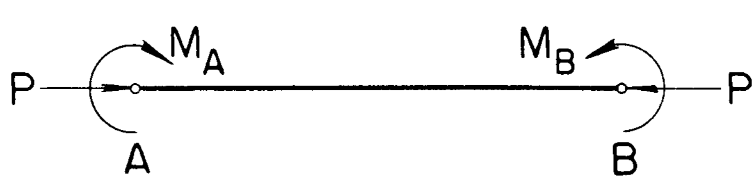


Fig. 3

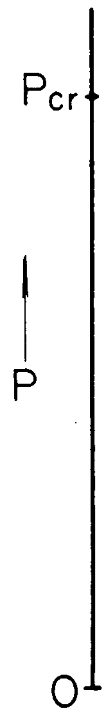


Fig. 4

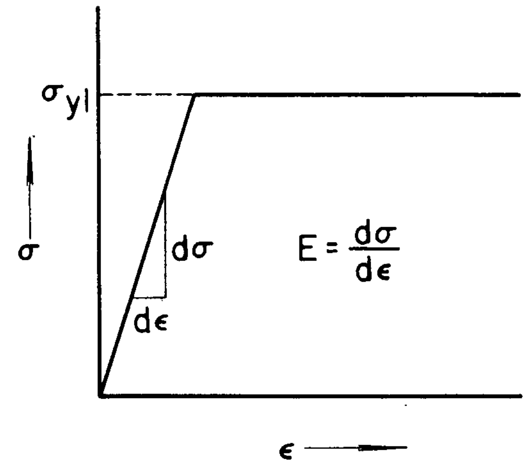


Fig. 7

Fig. 5

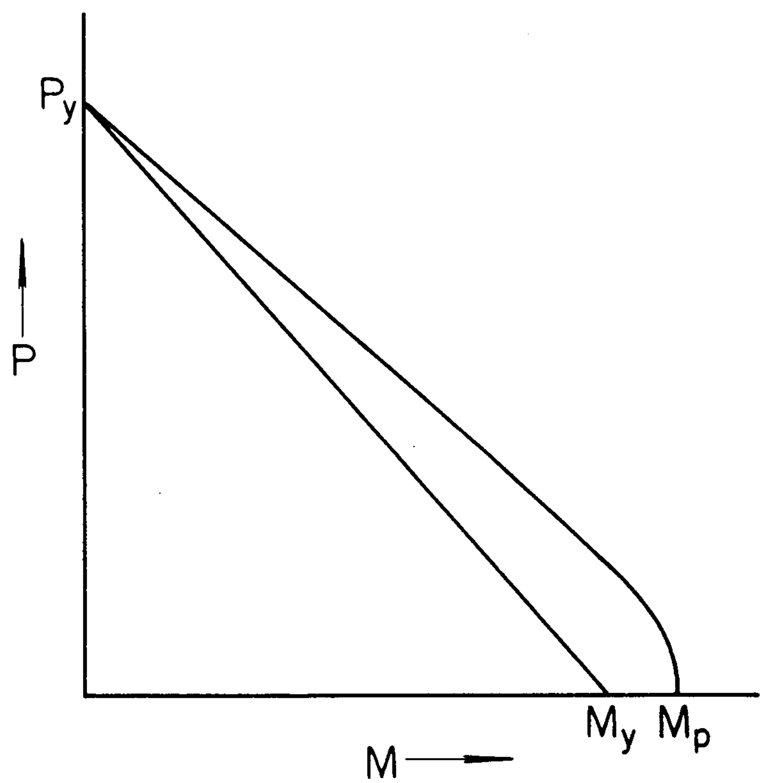


Fig. 6

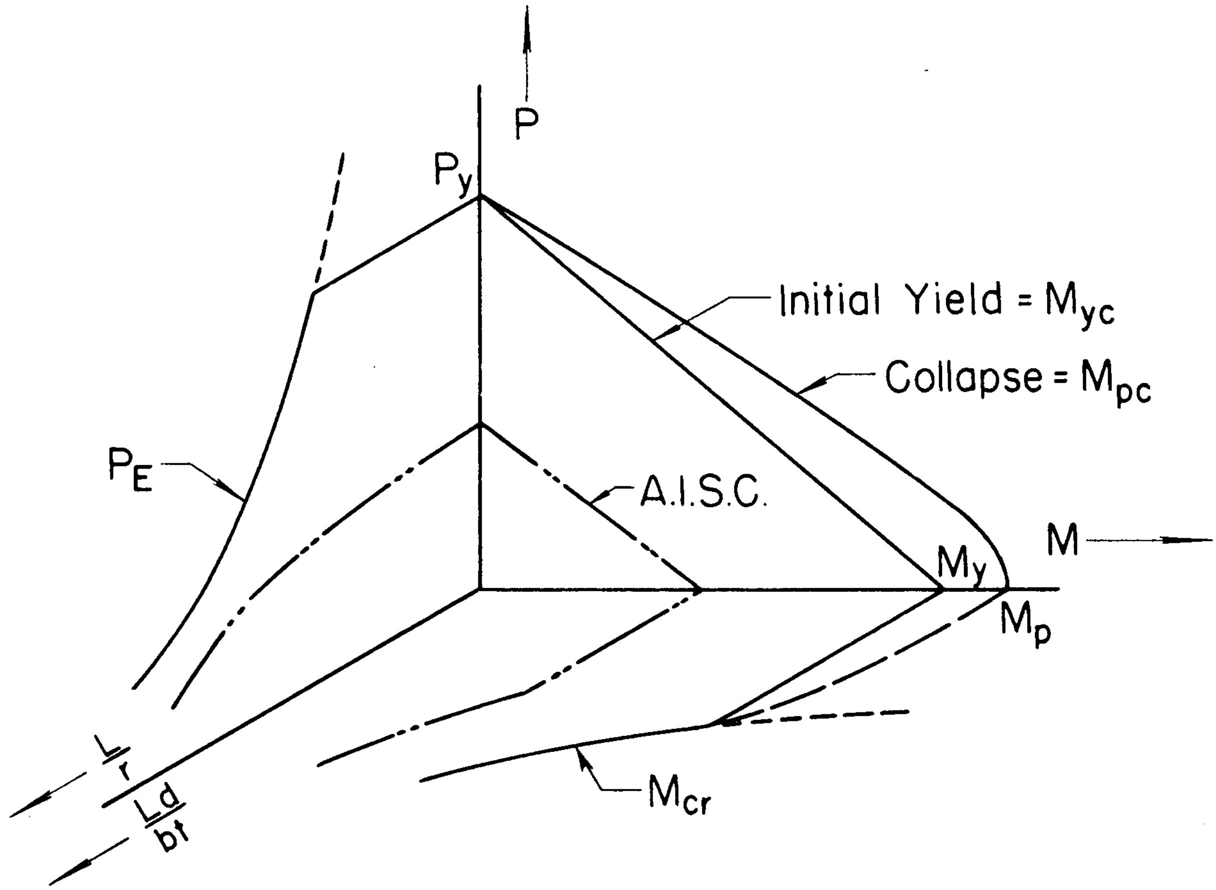

P
P_y
Initial Yield = M_{yc}
Collapse = M_{pc}
P_E
A.I.S.C.
M
M_y
M_p
M_{cr}
L/r
$\frac{Ld}{bt}$

Fig. 8

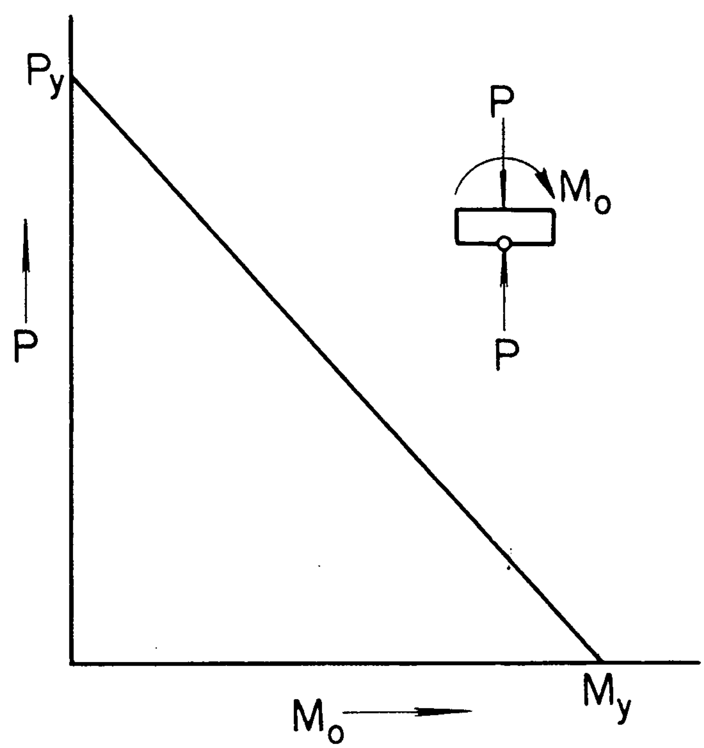

P_y
P
P
M_o
P
M_y
M_o

Fig. 9

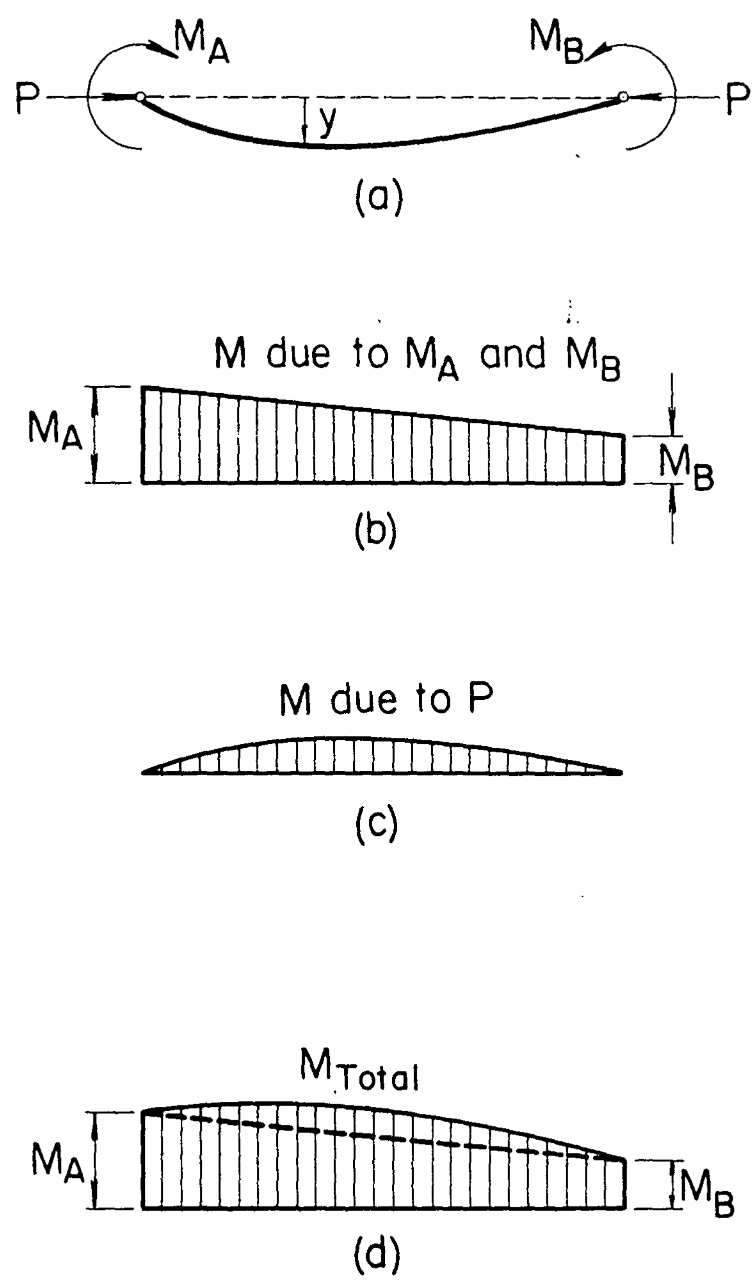
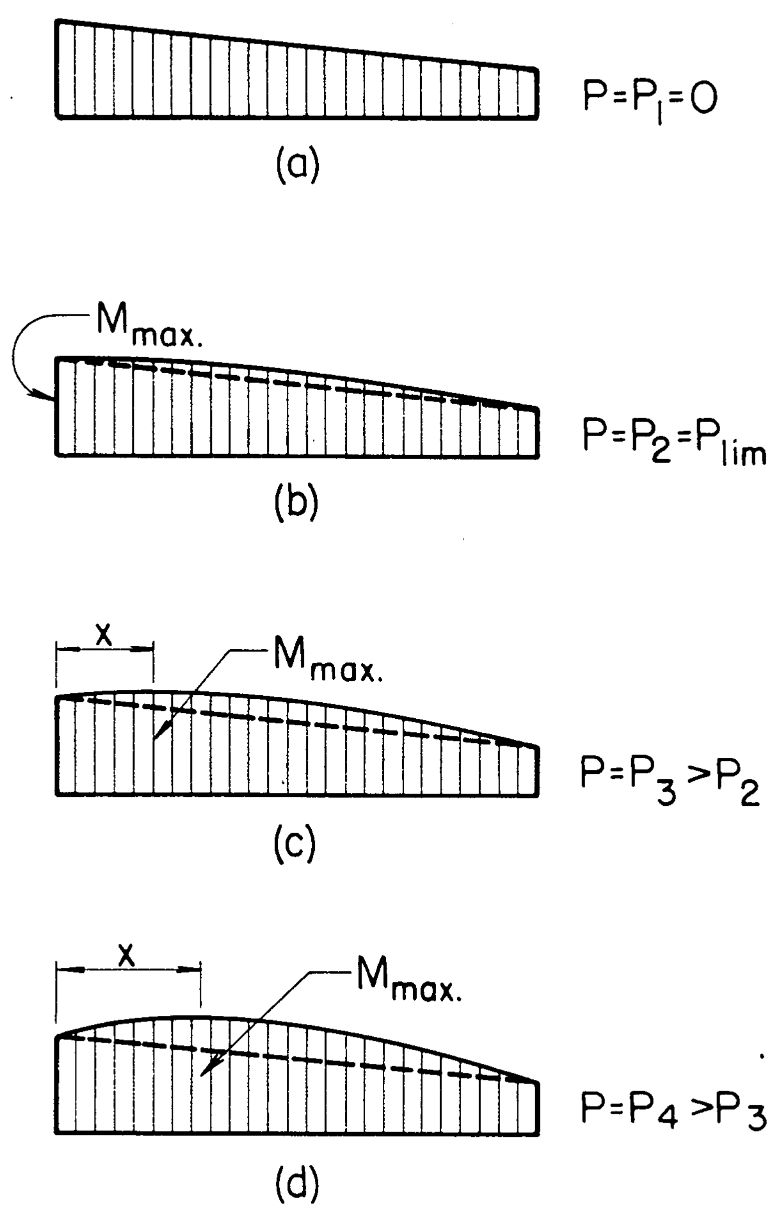


Fig. 10



where: $(P=0) < P_2 < P_3 < P_4 < (P_E \text{ or } P_y)$

Fig. 11

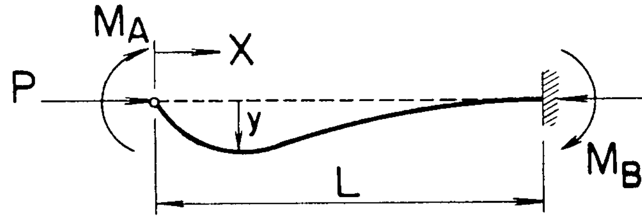


Fig. 12

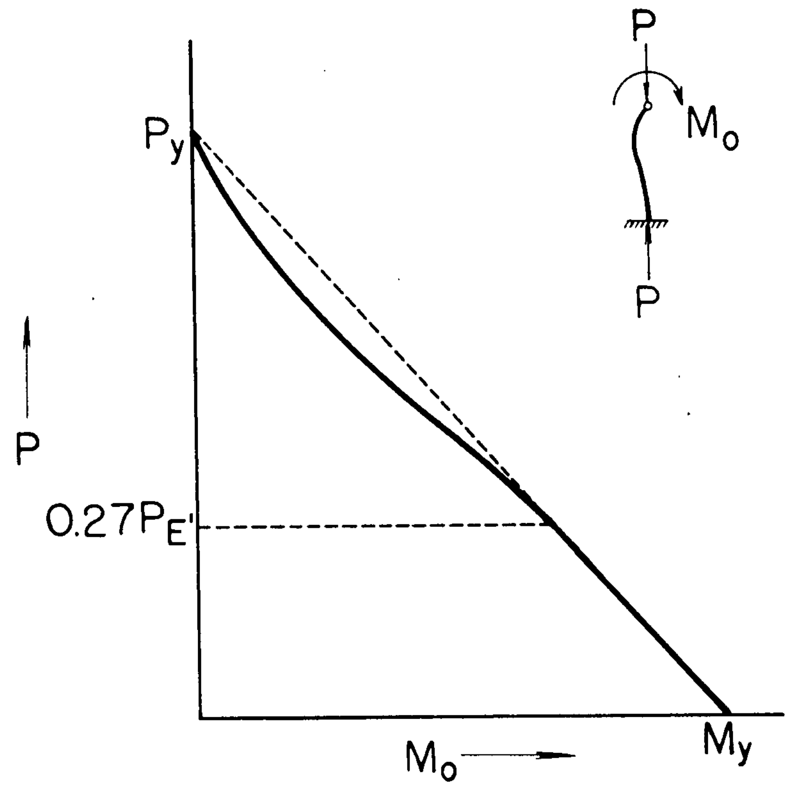


Fig. 14

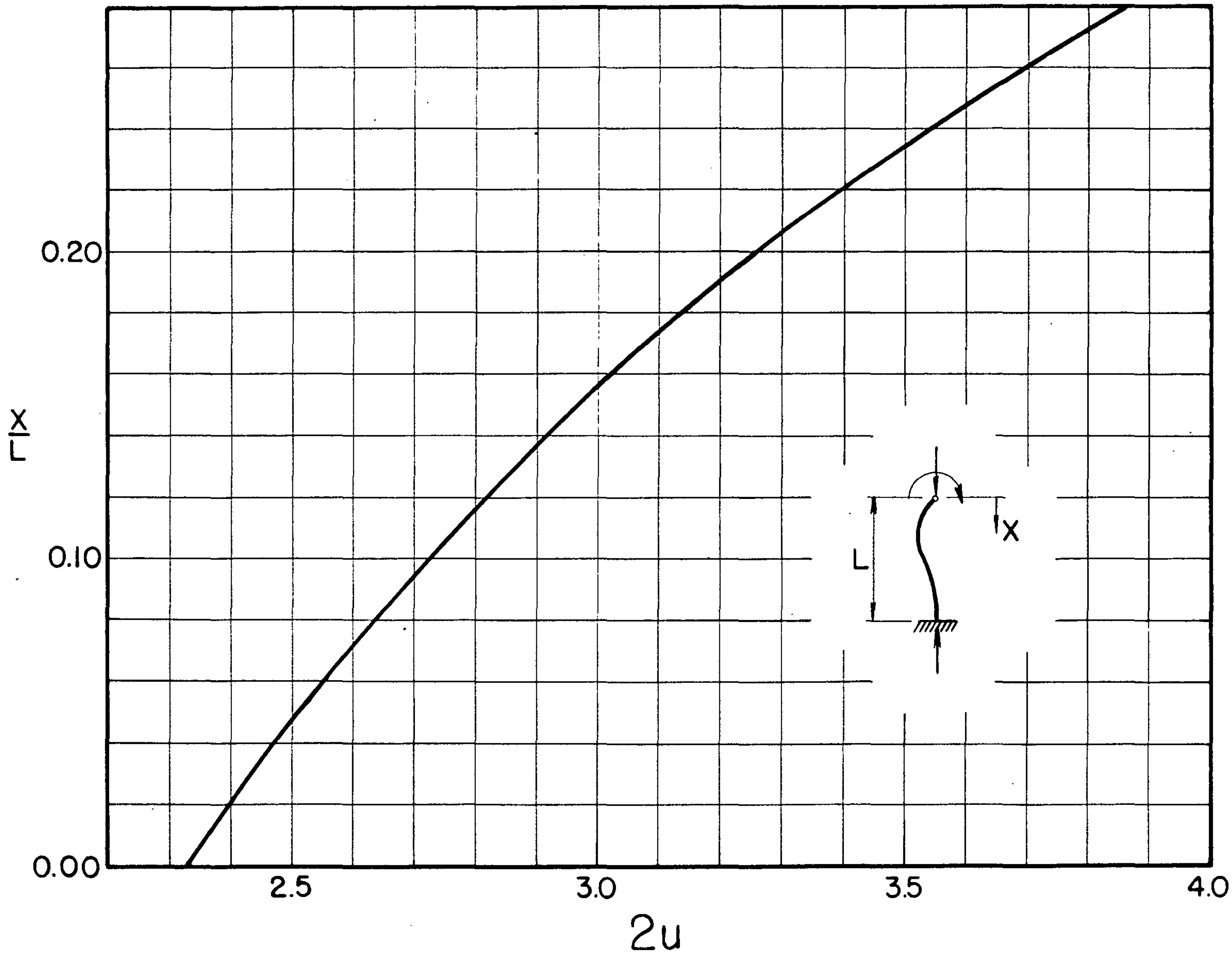


Fig. 13

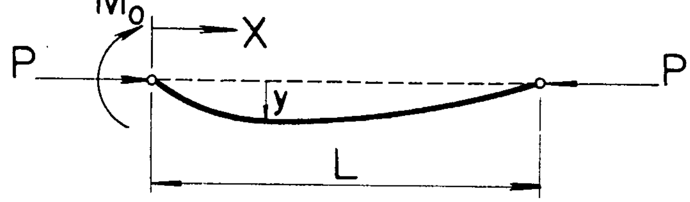


Fig. 15

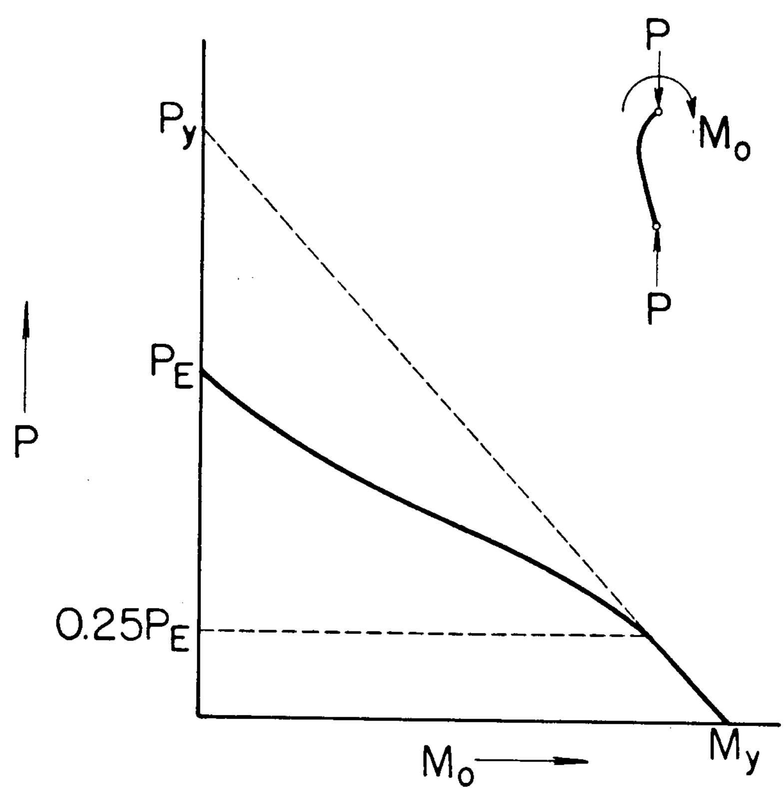


Fig. 16

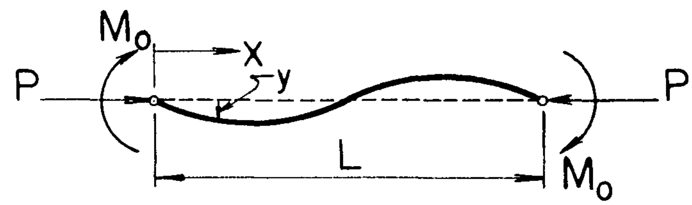


Fig.

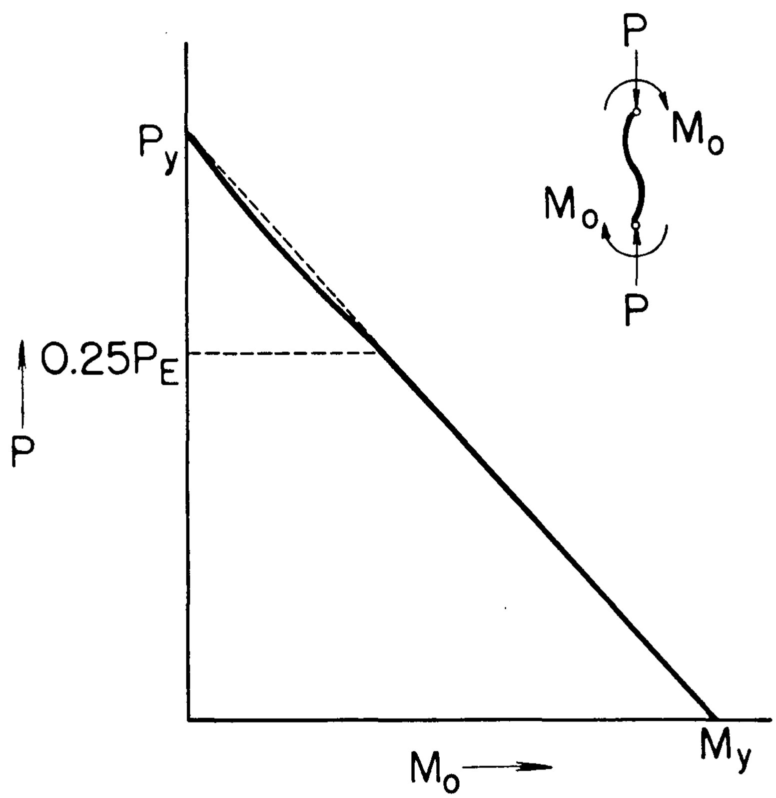


Fig. 18

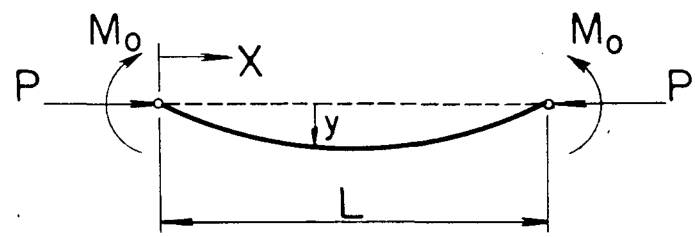


Fig. 19

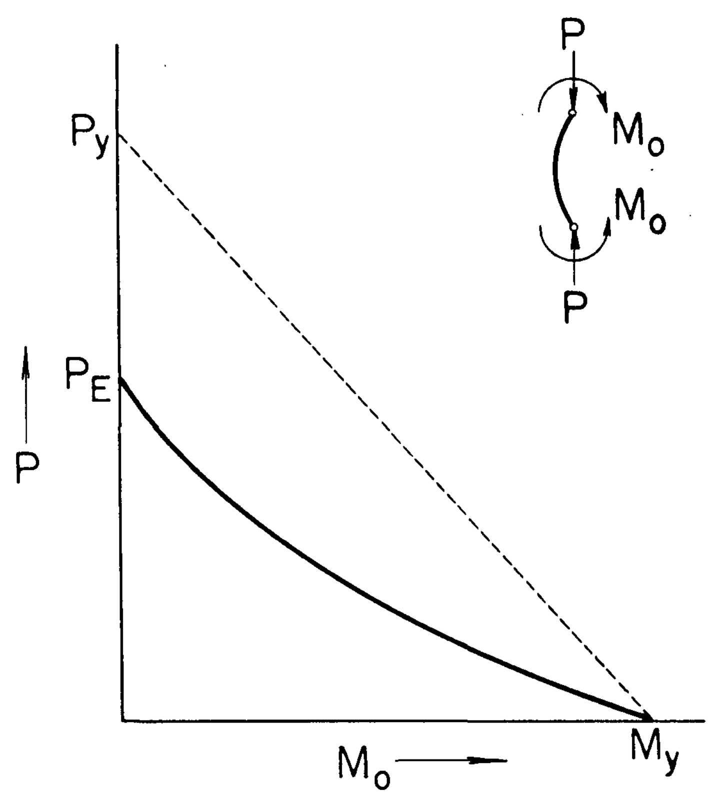


Fig. 20

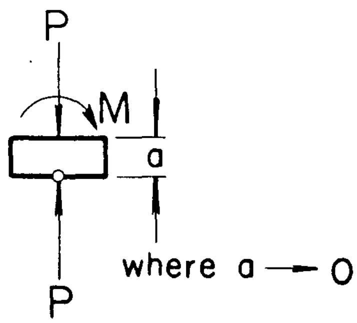


Fig. 21

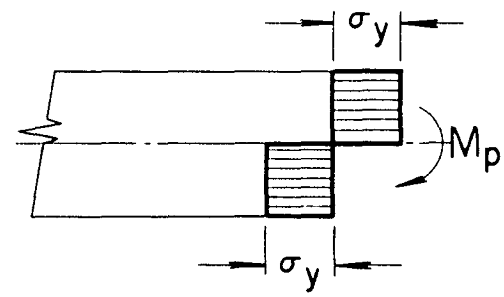


Fig. 22

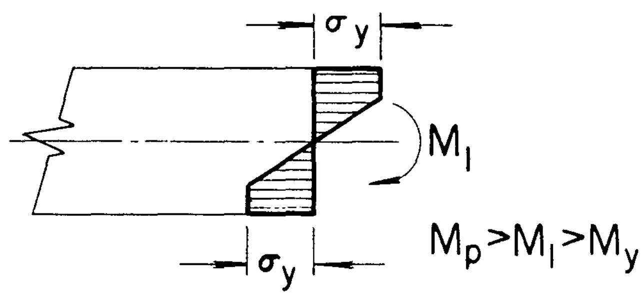


Fig. 23

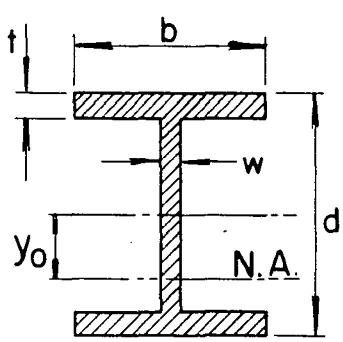


Fig. 24

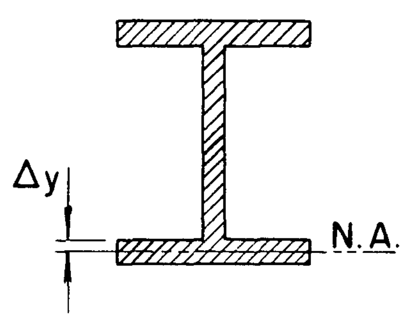
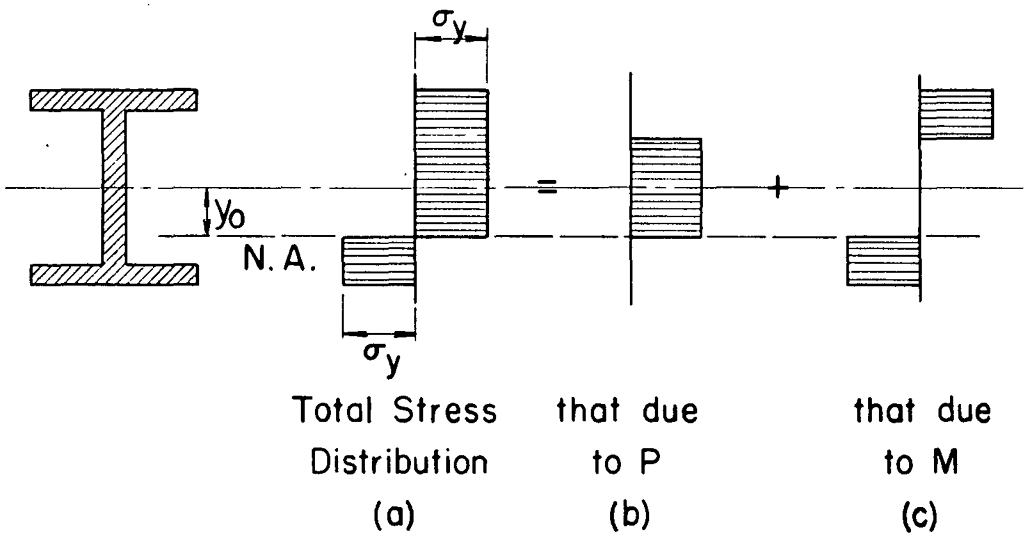
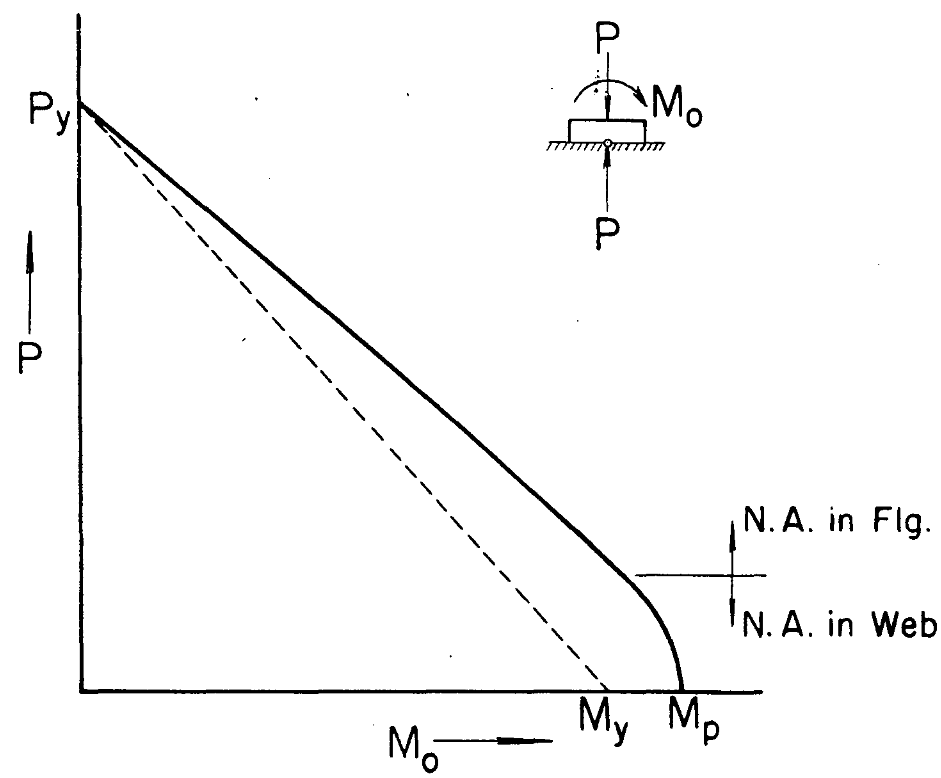


Fig. 26



σ_y
y_o
N. A.
σ_y
Total Stress Distribution
(a)
that due to P
(b)
that due to M
(c)

Fig. 25



P_y
P
P
M_o
P
N. A. in Flg.
N. A. in Web
M_y
M_p
M_o

Fig. 27

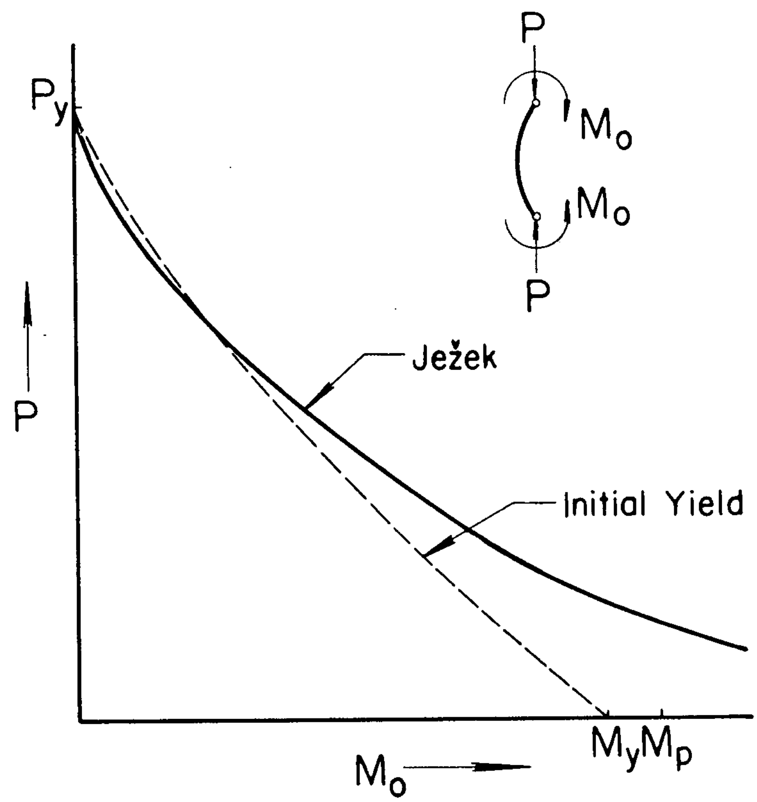


Fig. 28

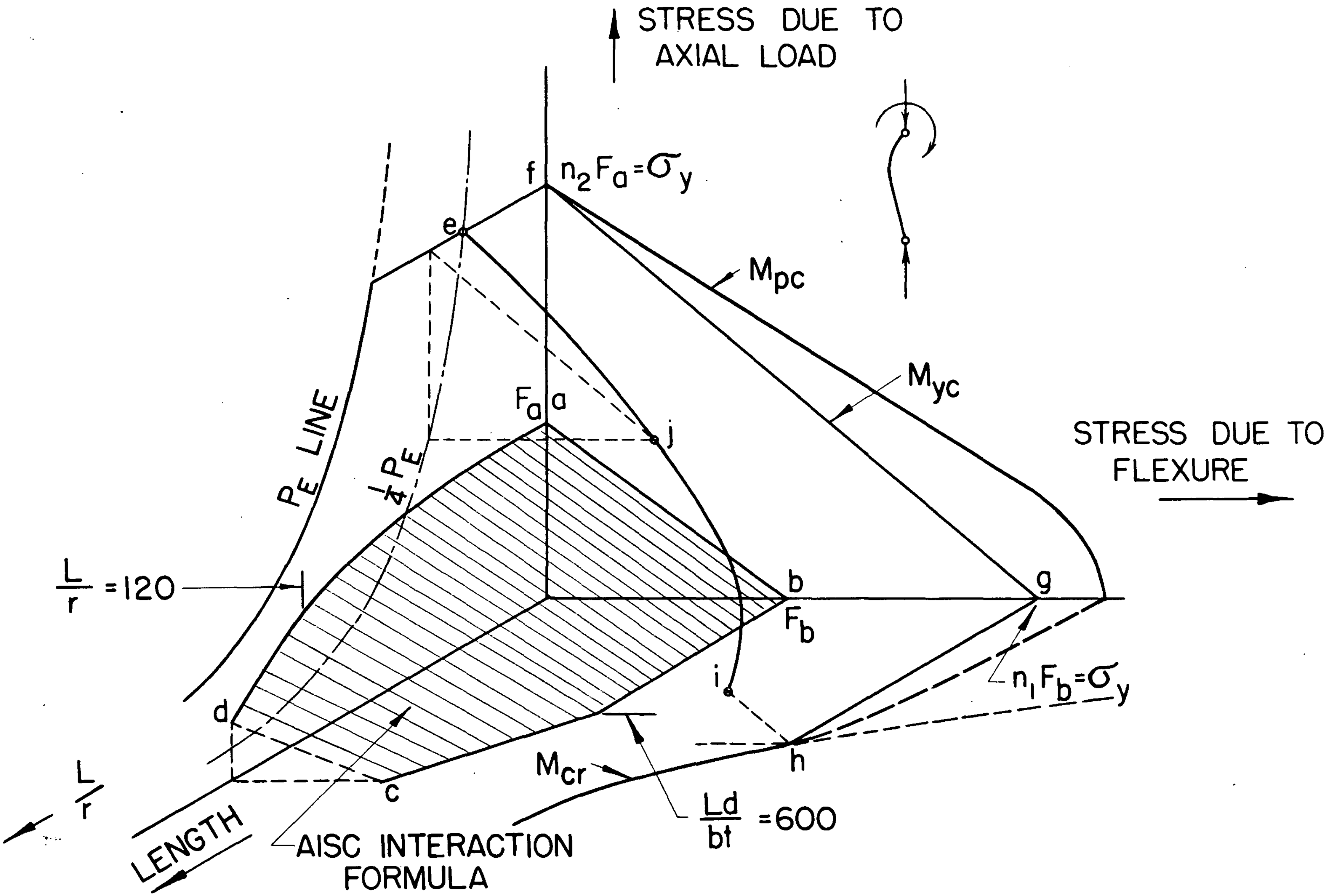


FIG. 29 LOADING CONDITION "d" THREE DIMENSIONAL INTERACTION CURVE

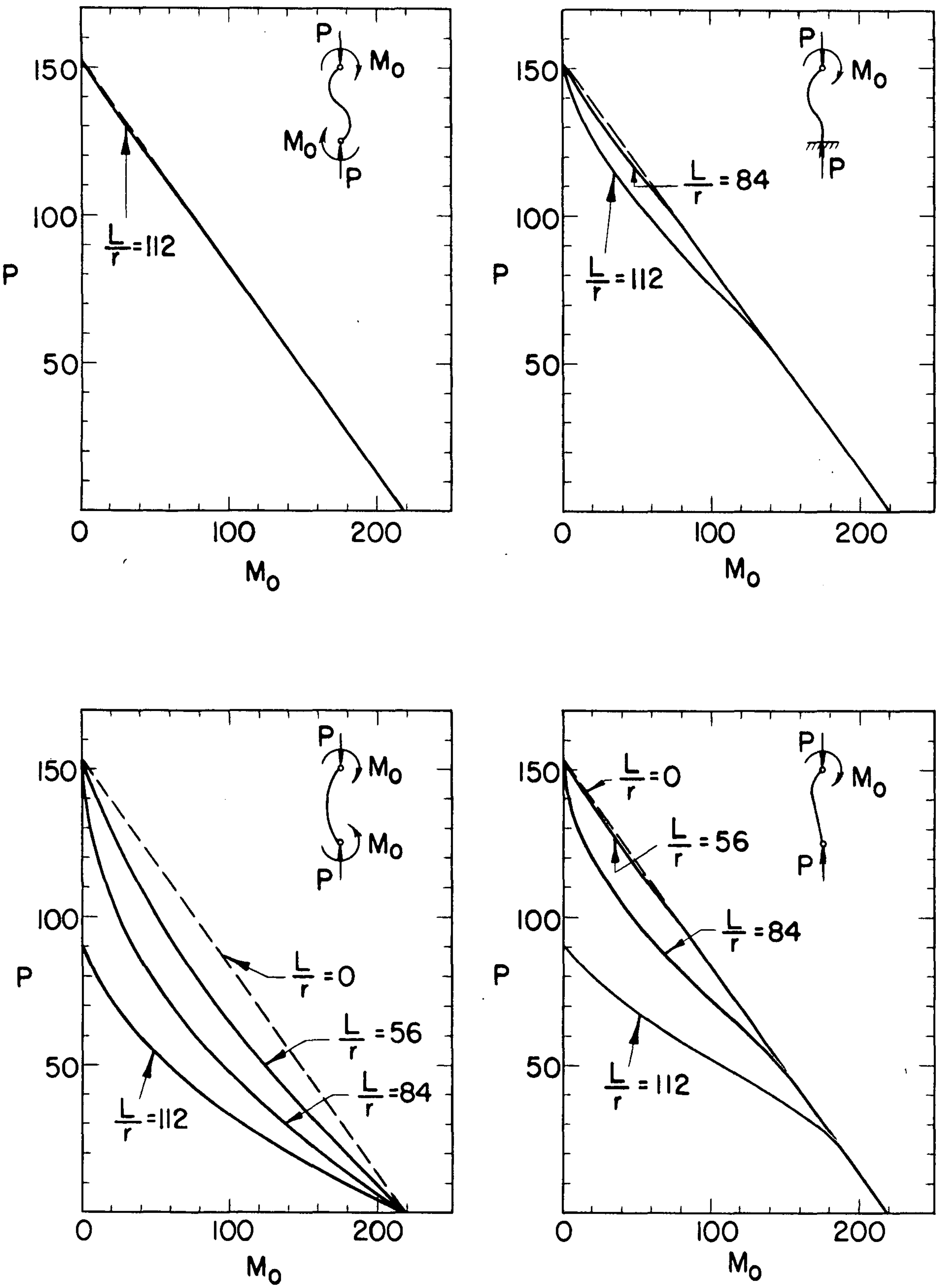


FIG. 30. INFLUENCE OF SLENDERNESS RATIO ON INITIAL YIELD INTERACTION CURVE

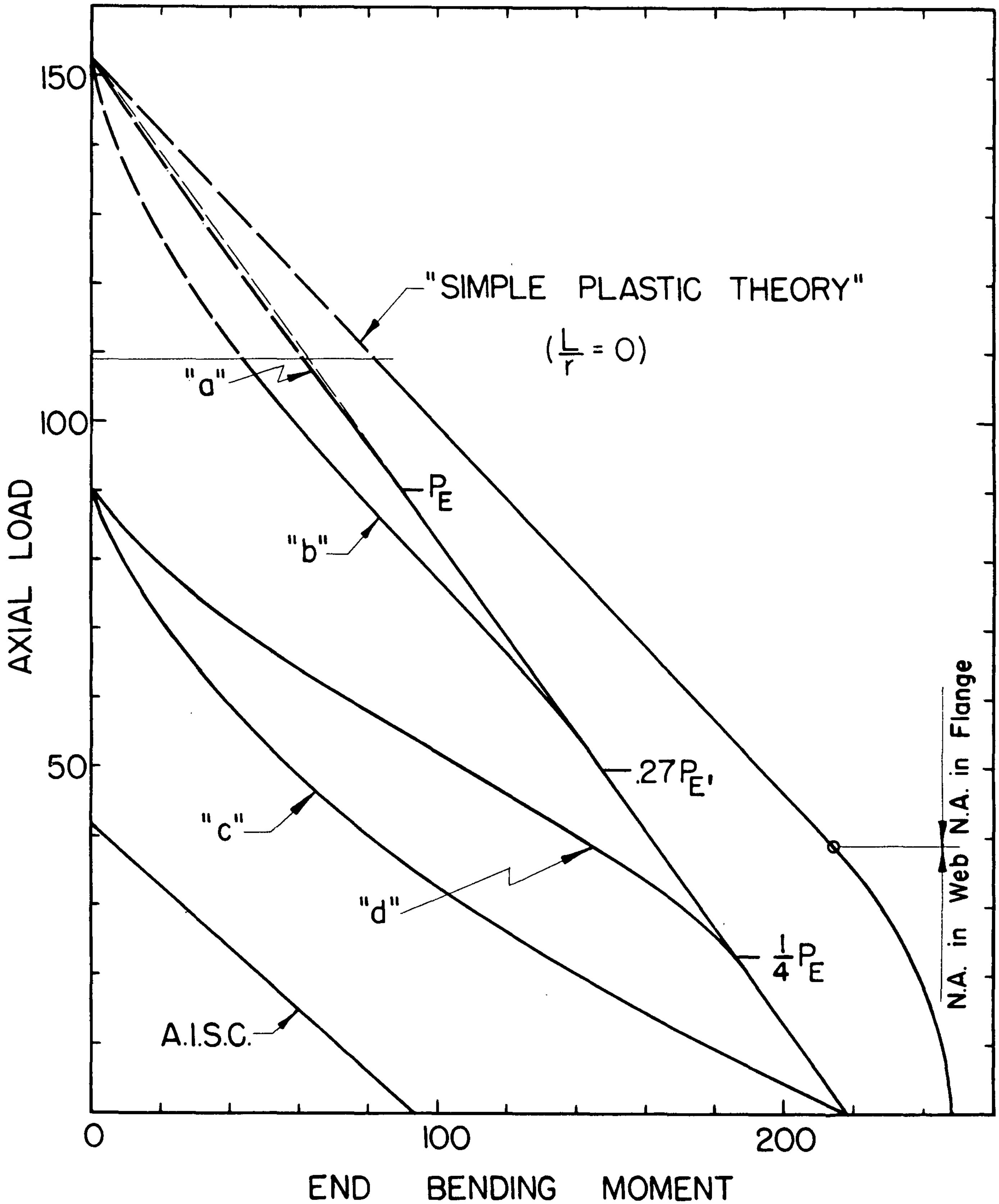


FIG. 31 INFLUENCE OF LOADING CONDITION ON INTERACTION CURVE

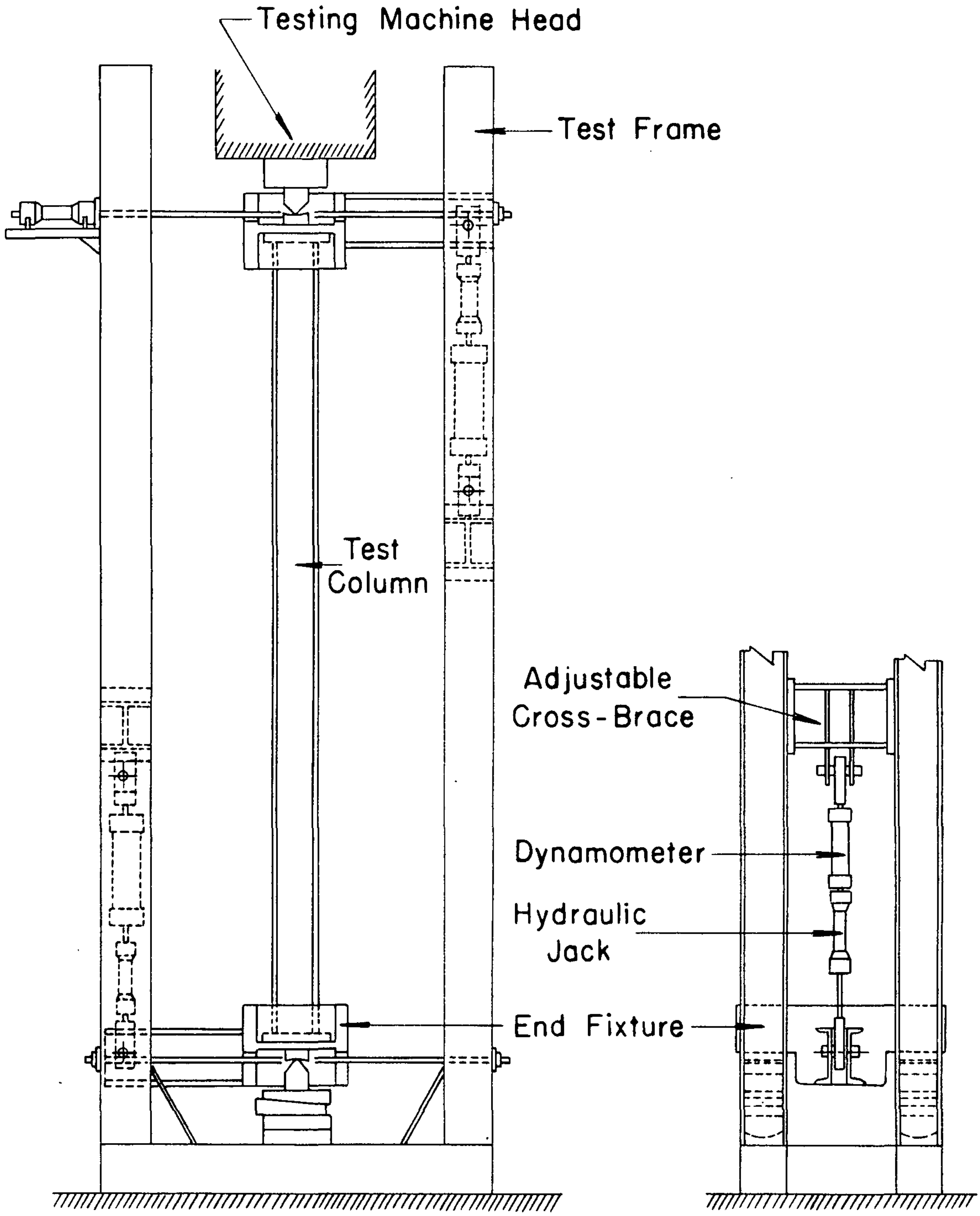

Testing Machine Head
Test Frame
Test
Column
Adjustable
Cross-Brace
Dynamometer
Hydraulic
Jack
End Fixture

Fig. 32

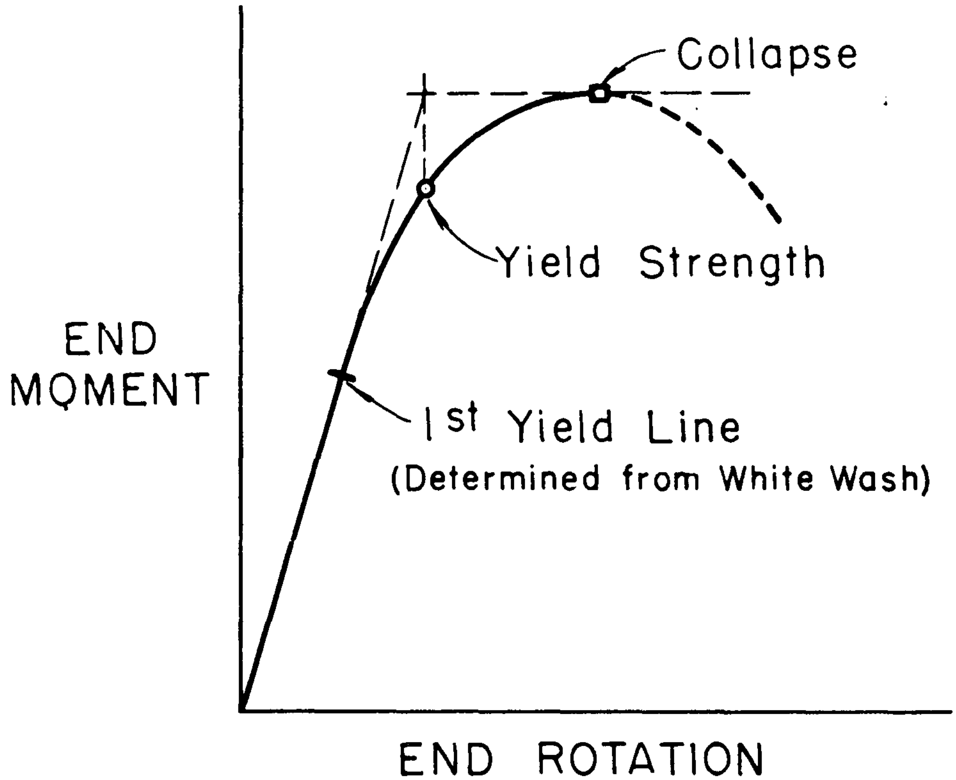
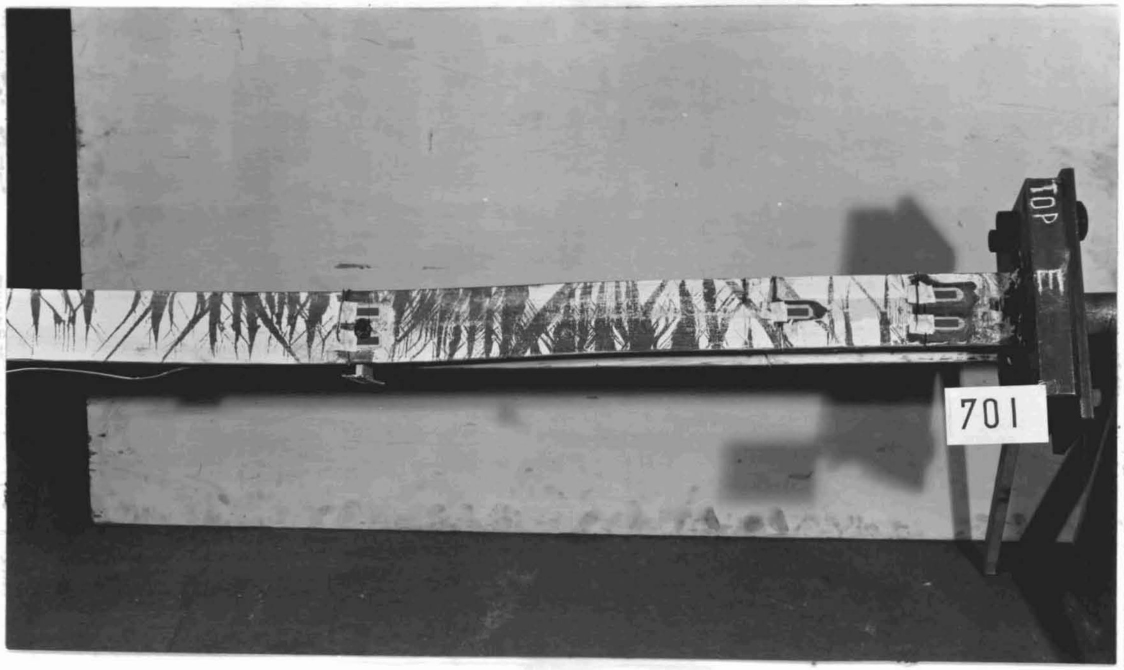

Collapse
Yield Strength
END MOMENT
1st Yield Line
(Determined from White Wash)
END ROTATION

Fig. 33



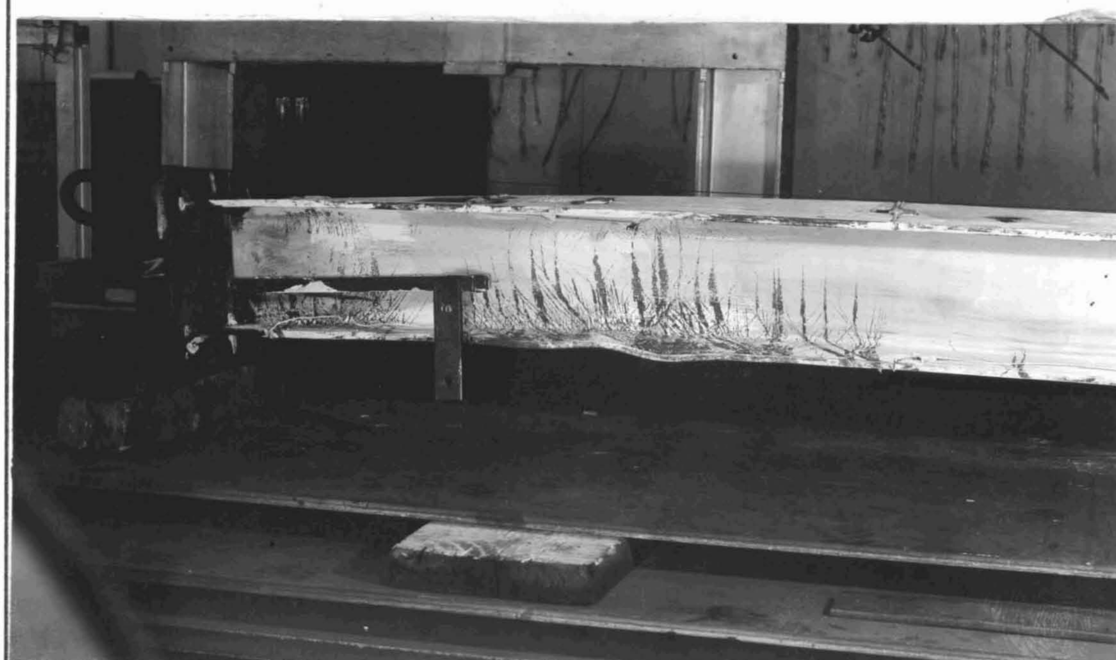


Fig. 42

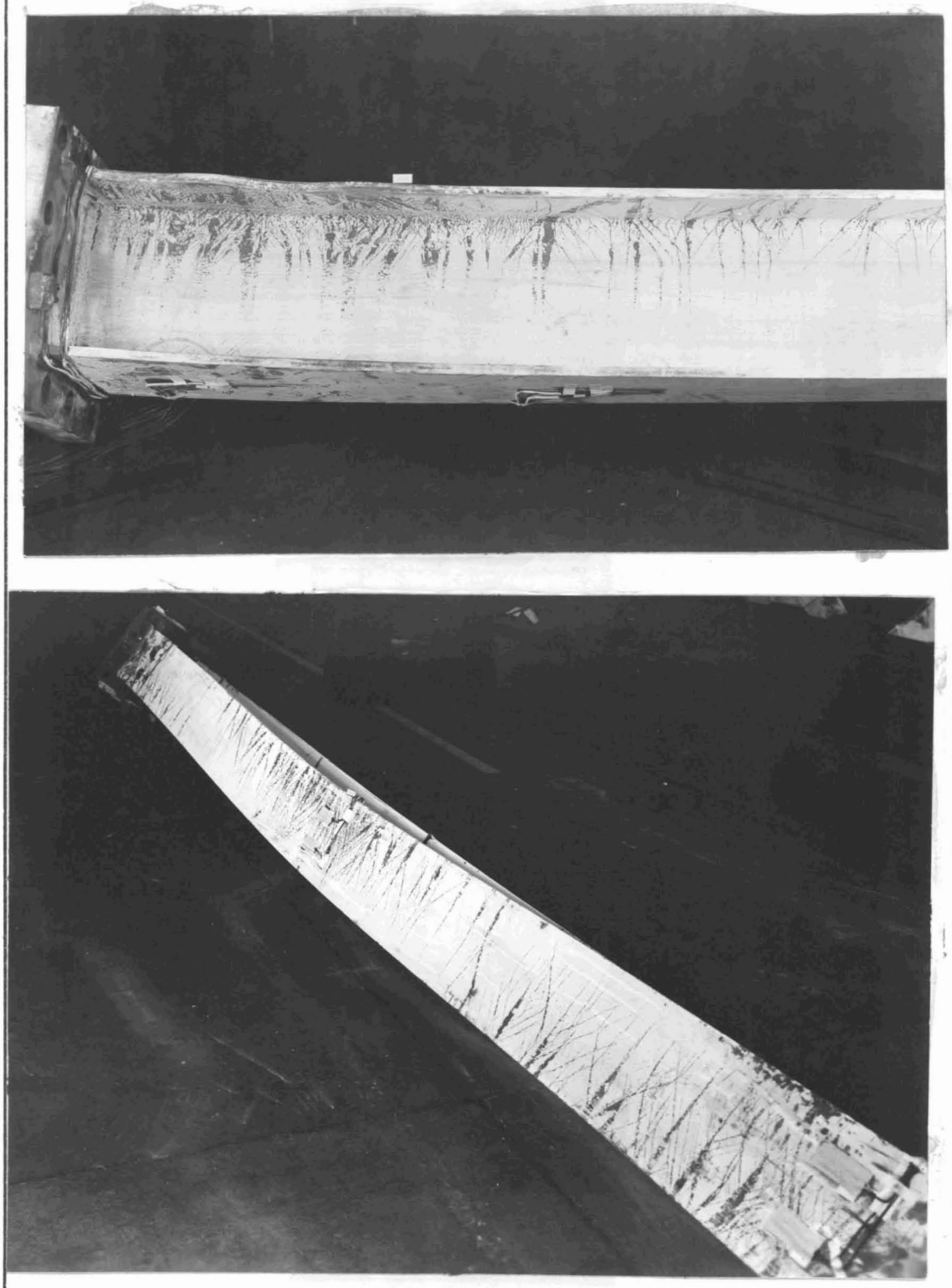


Fig. 43